Economies of Density and Congestion in Capital Rental Markets *

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Abstract

The development of rental markets for equipment is a potentially powerful mechanism to grant small scale producers access to capital and its technology. Governments in the developing world have recently engaged in stimulating rental markets through subsidies. These interventions have distributional effects as well as efficiency effects that are not well understood. This paper is the first one to assess the allocative efficiency of these markets in a developing economy. To do so, we combine a novel dataset of the universe of transactions from one of the largest providers of equipment rentals in India, our own census of farming households, a survey of detailed farmer characteristics and a structural model of frictional rental markets. Allocations are mediated by the distribution of service requests and providers in space and endogenous delays in service provision due to demand congestion. Small farmers are rationed out by market providers. A government subsidized first-come-first-served dispatch system benefits small-holder farmers through equipment access and declines in queuing time. However, this dispatch system entails equipment transportation costs that may well overturn the productivity gains to those farmers. Large farmers benefit from queuing with market providers and the benefit becomes stronger the larger the share of small farmers entering the rental market.

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1 Introduction

The development of rental markets for equipment is a potentially powerful mechanism grant small scale producers access to capital and its technology. Indeed, the history of the path of mechanization for currently rich economies suggests that equipment rental markets were a stepping stone to that process (Binswanger, 1986). But where contract enforcement is weak and overall wealth is low, rental markets may not emerge, and if they do, they may take a form that rations out productive and/or smaller farmers.

Governments in the developing world have recently engaged in stimulating rental markets by either subsidizing new capital purchases, or by subsidizing set up costs to the creation of these markets through public/private partnerships. These interventions have distributional effects as well as efficiency effects that are not well understood. What type of farmers get to access the market? and What returns do they get? and Was it optimal to subsidize equipment to begin with? This paper is the first one to assess the impact of such interventions. Fairness concerns may lead to servicing farmers in relatively less dense areas, incurring additional transportation costs from moving equipment in space; or whose productivity is low. If equipment purchases are subsidized, rental markets are more likely to arise for operations that do not need to be performed in all farms at the same time (Binswanger, 1986). But those operations may not be those with the highest returns. Due to the time-sensitivity of agricultural activities, equipment rental markets are likely to display congestion, because demand is synchronous and service capacity is fixed. By subsidizing increases in equipment supply the government could lower congestion, but it could also induce marginal farmers to demand services, slowing down service provision altogether. At the same time, it increases the incentives of larger land-owners to demand services from market providers, increasing their market power. The balance of these effects dictates the efficiency and distributional impact of the development of these rental markets.

Our paper combines the first available transaction level dataset of equipment rental markets in the developing world, a census of 130 villages characterizing the full demand for services, and a novel structural model of frictional rental markets to asses market efficiency and the distributional impact of government subsidies. We use data on the two prevalent types of provision of mechanization rentals. The first one is the provision of services by single owners of equipment, often farmers who own agricultural equipment and rent them out ("market" providers). Given service capacity constraints, these providers prioritize the service of large orders or orders in densely populated locations. Consistent with the data, this prioritization induces delays for farmers with relatively small orders or in lower density locations. The second one is a private-public partnership that set up custom hiring centers (CHCs) with the objective of providing machinery for small and marginal farmers. These providers allocate equipment on a first-come-first-served basis ("FCFS" providers). In this case, the order size and location is uncorrelated with the timing of service. Inframarginal farmers are driven into the government subsidize platform, but the additional transportation costs associated to moving equipment in space may well overturn the gains in productivity to those farmers. Through counterfactual exercises, we quantify the strength of these channels and argue that general equilibrium forces in prices and the equilibrium sorting of farmers to providers are quantitatively important to assess the allocative efficiency of rental markets.

We start by characterizing rental markets in Karnataka, India. We use detailed transaction data from the private-public partnership since 2017, including hours requested, acreage serviced, implement and location. This data is linked to our newly collected survey data on demographics, farmer assets, productivity and engagement in the rental markets.First, we document that requests for capital rental services are spread across space and that there is higher engagement in rental markets by large-scale farmers (more than 4 acres of farmland) than by low-scale farmers (cropping less than 4 acres). However, the distribution of these large and small farmers in space is such that once we control for equipment ownership levels in neighboring villages, there is no differential access between large and small scale farmers. Second, we document queuing for service fulfillment which varies throughout the season. This congestion effect is of first-order importance for the organic development of rental markets, and a key margin to assess the efficiency and distributional effects of the markets induced by the government subsidies. Third, because agricultural activities are highly time sensitive, the delay in the arrival of inputs associated to these queues is potentially costly. We quantify the costs of delays for productivity and profitability by estimating the optimal date for land preparation during a season. We choose land preparation because those are the processes for which the rental markets in Karnataka are most active. We compare profits for farmers that prepared the land away from that optimal date. We find that the average productivity cost for delay within a 10-day window of the optimal planting date is 8.5% per day.

To assess the merits of different dispatch systems for efficiency and productivity, we pose a novel structural model of frictional rental markets for equipment. There are two types of providers that differ in their order dispatch technology and face capacity constraints for service. There are two types of farmers that differ in their demand of equipment-hours and their locations in space. Their location induces costs in service provision because moving equipment in space is costly. Providers set rental prices and farmers choose in which provider to queue at, as in a standard directed search model. Consistently with the data, providers can serve up to three orders within a day, posing an interesting combinatorics problem given the provider's capacity. The main predictions of the model are that if small and large farmers are equally distributed in space, the efficient allocation is that small scale farmers only approach the FCFS provider, where their expected queueing time is lower. Large farmers should search for services both in the market and in the FCFS provider. However, their incentive to reach market providers increases with the share of small scale farmers in the market. The reason is that the queue length of large farmers at the market providers is lower. The cost of service is higher than from the FCFS, so that large farmers are indifferent in equilibrium. Disparities in the distribution of farmers in space have direct implications for the cost of provision.

To bring the model to the data we allow for additional heterogeneity in farmers equipment-hours requests and locations. We solve and estimate our structural model of optimal dispatch, using administrative data from the FCFS platform and the Census data for market providers and our survey data on farmers' productivity.The link between equipment-hours demanded, productivity and plot location is inferred directly from the data. Through a simulation exercise, we find that the government subsidized provision, which relies on a first-come-first-served dispatch, benefited small-holder farmers through declines in queueing time. However, this dispatch system entails equipment transportation costs that could overturn the gains to those farmers. Large farmers face lower delays when renting from market providers, and their location is such that when market providers minimize transportation costs, larger farmers disproportionally benefit from queuing with them.

A unique dimension of our problem is that location-specific demand congestion generates losses in productivity due to delays in service provision. Congestion is endogenous to the queueing and pricing decisions of market participants. We propose a tractable framework to study congestion through a queueing service model. We expand the directed search environment in Shi (2002) to allow for provision of multiple orders and service capacity constraints.[1] Delays are often overlooked as a barrier to technology adoption, yet is a potentially important mechanism in sectors like agriculture, where returns are extremely sensitive to the timing of farming activities. Delays are typically the result of synchronous queuing.

The notion that there might be scale economies associated to concentrating production in certain locations is explored by Holmes and Lee (2012) in the context of crop choices of adjacent plots. Contemporaneous work by Bassi et al. (2020) document the workings of rental markets for small carpentry producers in urban Uganda and argue that frictions in their setup are relatively limited. Distinctively, agricultural equipment needs to be moved in space to reap its benefits and the time sensitivity of agricultural production makes demand synchronous. We show that these two margins are key determinants of the efficiency of usage of capital services through the rental market, and its distributional and productivity impact.

The role of geography for the efficient allocation of factors of production in agriculture has been studied by Adamopoulos and Restuccia (2018). In their paper they focus on the characteristics of the soil and the potential yields of different plots to assess the degree of misallocation of factors of production. We instead consider the effects of congestion in capital service provision (allocation) due to the potential spatial misallocation of service providers and demand for those services. Spatial misallocation has been studied in the context of the rural-urban gap in labor income by Gollin et al. (2013); Lagakos et al. (2015).[2] In this paper we focus on the distributional effects across land-sizes and locations of equipment rental markets.

This paper also relates to the literature that emphasized barriers to adoption of technology in agriculture (Suri, 2011) and of mechanization in particular: Pingali et al. (1988) highlighted the role of contractual enforcement problems while Foster and Rosenzweig (2017) emphasizes economies of scale. Small relatively poor farmers, do not find it optimal to adopt capital intensive practices when they entail the purchase of equipment, a relative large expense whose services are used for a limited period in the season. Importantly, small farms

[1]As highlighted by Lagos (2000) and Sattinger (2002), queueing models are powerful to micro-found a matching process between, in this case, farmers' orders and service provision.

[2]Spatial misallocation has been studied in the context of taxation across US states by Fajgelbaum et al. (2018), while Hsieh and Moretti (2019) study misallocation of workers across MSAs.

are prevalent in poorer economies Gollin et al. (2013); Adamopoulos and Restuccia (2014) where the incidence of technology in agriculture and overall productivity is low. Indeed, the relative costs of capital to other factors of production such as labor costs have been emphasized as deterrent for the adoption of technology, (Pingali, 2007; Yang et al., 2013; Yamauchi, 2016; Manuelli and Seshadri, 2014). In this paper we focus on the role of geography and density in assuring access to capital services through rental markets.

2 Equipment rental markets for small-holder farmers.

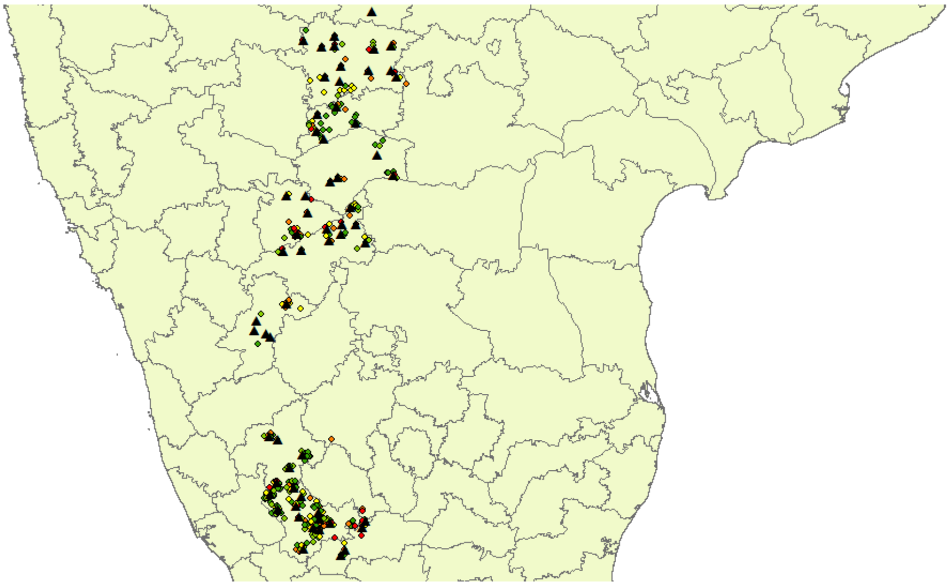
2.1 Data description

We study agricultural rental markets in rural India. Rental markets for equipment have been expanding rapidly in the development world exploiting new connectivity technologies and renewed governmental interest in subsidizing the use of mechanized practices. We collected data from on of the largest providers of equipment rentals in India and document salient features of the working of this market. The study focuses on the state of Karnataka, one of the least mechanized states in India (Satyasai and Balanarayana, 2018), and where the majority of agriculture is on small farms (less than 4 acres). Our own census of farming households in 150 villages across the state suggest that equipment ownership rates are low and that farmers rely on informal equipment rental markets within each village.

In 2016, the state government partnered with the largest manufacturer of agricultural equipment in India to design and manage a platform through which farmers could rent equipment. They generated a subsidy scheme for equipment purchases to create custom hiring centers (CHCs, also known as "hubs") in 25 districts throughout the state. Currently active hubs are plotted in space in the left panel of Figure 1. These CHCs provide rental services in nearby areas and farmers can access these services through a call-center, via an app on a smart-phone, or in person at the CHC.

Our first source of data comes from the universe of transaction-level data maintained electronically by these hubs. This administrative data consists of all the transactions completed through that platform since 2016 for about 60 hubs in Karnataka (27 hubs entered in 2016, 29 in 2018 and 10 in 2019). Over the time period covered by the data (October 2016-

Figure 1: Locations of CHCs and Demand



Triangles indicate CHCs. We aggregate demand following the village where each farmer is registered. Green dots correspond to demand for the smallest plots (1 acre or less), red dots correspond to demand for the largest plots (4 acres or more).

May 2019), over 17,000 farmers from 840 villages rented equipment from these CHCs. The data contains information on number of hours requested, acreage, implement type, as well as farmer identifiers (such as their name, village, and phone number). Equipment available varies across CHCs (hubs) but the median hub provides equipment that ranges from sprayers to Rotavators or ploughs. Rental prices in the platform are about 10% below market prices on average, as part of the government's initiative to increase mechanization access. The service provision is first-come-first-served. When a service is fulfilled, a professional driver brings and operates the equipment at the farmer's plot. Equipment arrives within a lapse of two days in most cases. If delays are longer, farmers are informed at the moment of booking (and may choose to cancel). Table 1 reports the 10 most commonly rented implements for the years 2017 and 2018, the number of transactions recorded for each implement, their per-hour rental price, and month where the implement is most commonly rented (has the highest number of transactions).

We use two additional sources of data. The first is survey data collected in June-July 2019 around a representative sample of the equipment hubs in the main sample. The survey includes approximately 7,000 farmers, and asks for detailed information on input and output

Table 1: Summary Statistics of Commonly Rented Implements from Rental Database

	Commonly Rented Implements		
	Number of Transactions	Median Price Per Hour	Peak Month
Rotavator 6 Feet	11,239	770	July
Cultivator Duckfoot	7,287	550	April
Cultivator 9 Tyne	5,245	525	May
Plough 2MB Hydraulic Reversible	3,716	450	February
Trolley 2 WD	2,436	250	January
Harvester Tangential Axial Flow (TAF)-Trac	2,048	1800	May
Rotavator 5 Feet	1,811	700	September
Blade Harrow Cross	1,793	360	March
Knapsack Sprayer 20 Litres	1,688	22.5	October
Blade Harrow 5 Blade	1,600	360	June

per plot which allow us to generate measures of productivity at the plot level. We build an inventory of the assets available to the farmer. We also ask about their engagement in rental markets, rental prices, and perceptions on barriers to participation in the market.

The second is the ICRISAT household-level panel data, which contains detailed agricultural information, including, season-level data on agricultural operations, their timing, costs and total revenues. The data covers eighteen villages over 2009-2014. The villages are from five Indian states - Andhra Pradesh, Gujarat, Karnataka, Maharashtra, and Madhya Pradesh.

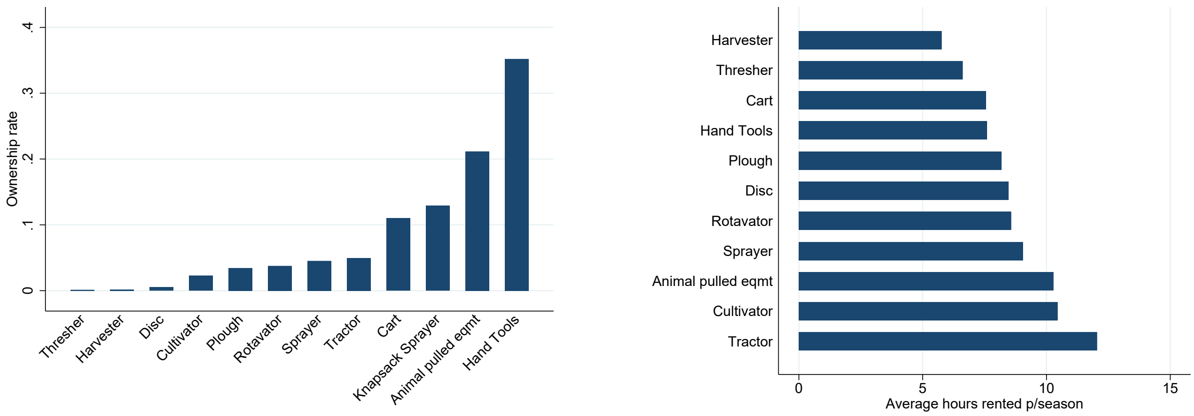
2.2 Motivating facts

We start by describing the characteristics of the service demand and farmers equipment supply. Then, we focus on a handful of outcomes that are informative to the theory of frictional rental markets that we describe in Section 3. First, because agricultural activities are highly time sensitive, the timing of demand is synchronous leading to endogenous waiting times as a function of service capacity. The service capacity includes farmers' ownership as well as CHCs capacity. Second, because equipment needs to travel for transactions to take place, the joint distribution between travel times and the scale of demand, i.e. equipment-hours per request, is a key input when optimizing service provision. Third, delays in service provision are costly to farmers, because they affect field productivity. In what follows we document each of these three features.

2.2.1 Service Capacity and Service Demand

We start by reporting patterns of ownership (service capacity by farmers) and rentals of equipment across the farmers in our survey (see Figure 2).[3] Most farmers report to own hand tools and animal pulled equipment. Less than 10% of the farmers report to own larger equipment such as tractors, or rotavators and cultivators. At the same time, tractors and cultivators are among the pieces of equipment with the highest equipment-hours rented. The average hours rented in a season per farmer is 12 hours for tractors and 10 hours for cultivators. These rental transactions mostly engage relational contracts. We collect information on the typical customer for a farmer that rents out his/her equipment. We find that 72% of owners report to rent out to people they know from the village or who they have worked with in the past.

Figure 2: Ownership and rentals by implement.



Owned   Rentals

The ownership rate is the share of farmers that report to own a given implement relative to the total population surveyed. Rental hours correspond to the average hours reported for the whole season.

Given the disparities in value of agricultural implements as well as their contribution to production, it is useful to construct a measure of efficiency units of equipment services from rentals and owned equipment. That is, we measure efficiency units of equipment by the product of average hours of usage during a season h_i, market rental prices, r_i and the number of pieces of equipment of implement i owned or rented, N_i. Hence, we define efficiency units

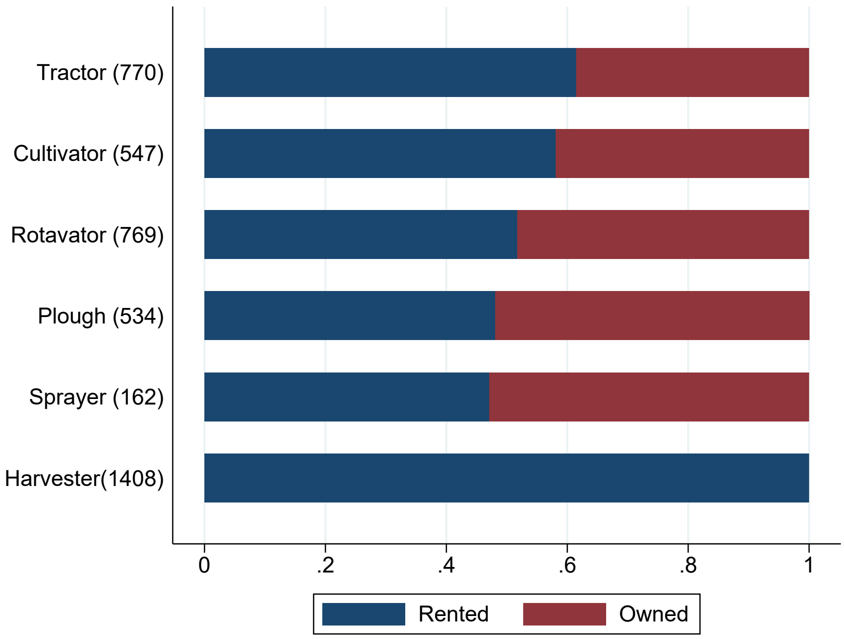
[3] Appendix D reports similar statistics using data from the Census.

of equipment in a farm k as

$$k = \sum_i N_i r_i h_i$$

The main hypothesis behind this measure is that rental rates shall reflect differences in the efficiency provided by a good with more expensive equipment providing higher efficiency units into production. The main challenge in constructing such a measure is the availability of market rental prices. We exploit our transaction level dataset to construct mean rental prices per implement at the village level. Figure 3 displays log owned and rented services. Harvesters (the most expensive implement in our bundle) is reported to be only rented. For those farmers using tractors, more than 60% of the services available in the farming sector come from rentals whereas the remaining 40% stem from ownership. Services associated to smaller and cheaper equipment, such as sprayers, are equally accounted for by rentals and ownership.

Figure 3: Capital services from ownership and rentals, by implement.



Shares of log capital services by implement and ownership/rental status. Average rental prices for an hour of service (in ₹) are reported next to each implement.

2.2.2 Queuing and disparities between large and small holders

The demand for equipment rental services vary by agricultural process and therefore throughout the agricultural season. The synchronous nature of many of these processes across farmers induces queuing in the market. Our transaction level data allows us to measure demand fluctuations by computing hours outstanding for service at a daily frequency. We focus on two commonly rented implements for land preparation, rotavators and cultivators. Indeed, our survey data indicates that farmers are most likely to engage in the rental market for land preparation.

Figure 4 shows hours of unfulfilled orders for each of these implements over the 2018 kharif season. Queueing peaks by the end of July for rotavators and beginning of August for cultivators. At the peak of the season, the average provider faces 40 hours of demanded services in queue, which account for over 12 orders at a point in time.

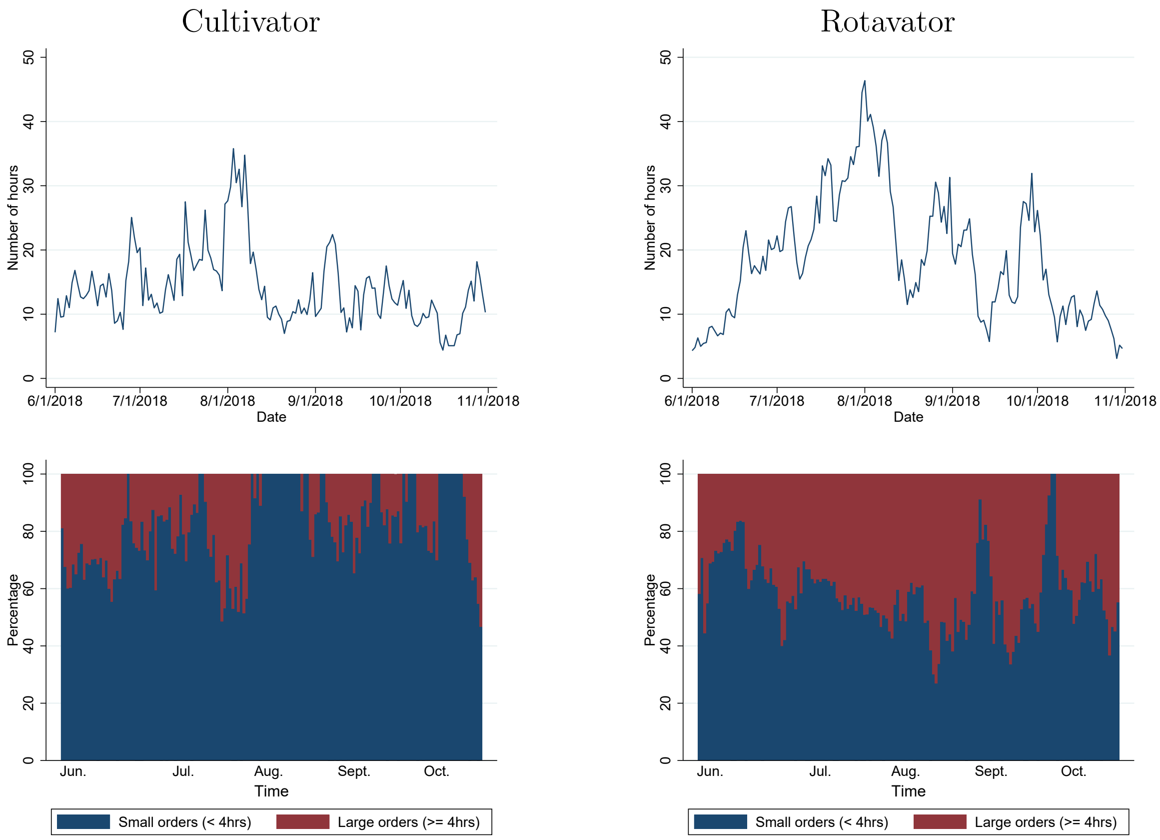
Demand moves distinctively between large and small requests, measured in service hours (Figure 4). A large portion of hours outstanding are accounted for by small orders (less than 4 hours of service), although at pick time the share of hours accounted for by large and small holder farmers equalizes. This is not explained by a higher number of large orders but rather by larger orders altogether.

As demand fluctuations over the season in a somehow predictable manner, it is expected that service supply may adjust. To explore these movements we compute service rates as the fraction of serviced hours within a day divided by the number of service hours outstanding. On average, we see up to 3 orders being fulfilled during a day per equipment piece. We find that service rates move during the season, and that they positively correlate with hours demanded (see Figure 5. However, they do not move enough to avoid queueing and congestion in service provision, likely due to capacity constraints.

2.2.3 Spatial distribution of rental services

We first document that service delays are negatively associated with cultivated size suggesting that even if the productivity costs of delays are of same magnitude between small- and large-holder farmers, the incidence of those delays is disproportionally bear by those with small plot sizes, columns (1) and (3) in Table 2. It is possible that these delays are explained

Figure 4: Hours outstanding in the queue.



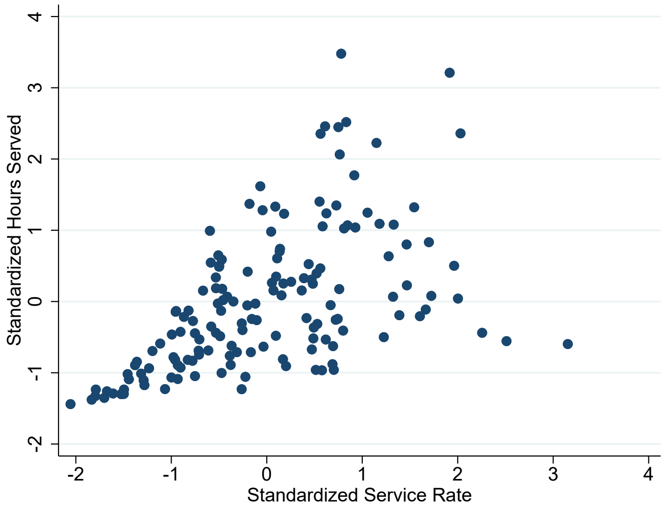
by size

by size

Averages hours outstanding in the queue across hubs in Kharif 2018, overall (top panel) and by order size (bottom panel).

by the geographical location of plots since equipment needs to travel to generate services. Columns (2) and (4) Table 2 show that delays have an important spatial dimension–adding village fixed effects substantially attenuates the coefficient on the log of land size, and increases the r-squared by eight or nine times (depending on whether only positive delays are considered, or all delays are included in the regression). That is, in the surroundings to a particular village, small and large farmers face similar delays, but if this clustering is not accounted for, smaller farmers face longer delays.

Figure 5: Service rates.



Averages across hubs (rotavator) Kharif 2018.

Table 2: Delays as a Function of Land Area and Location Fixed Effects

	Delays (Sum of Average Delays Over the Season)			
Log(Area)	-0.215*	-0.144	-0.319**	-0.128
	(0.115)	(0.0926)	(0.145)	(0.108)
Observations	5,615	5,615	4,345	4,345
R-squared	0.002	0.182	0.003	0.252
Village Fixed Effects	No	Yes	No	Yes
Mean Delays	2.158	2.158	2.789	2.789

Estimated coefficients from a regression of reported delays in service provision and the log(area) owned. The first two columns include those that report zero delays whereas the last two columns only focus on those that report positive delays.

2.2.4 Impact of Delays on Productivity

Using data from our own survey of farmers we find that farmers list service delays as their main problem when renting equipment, and that delays are more prevalent than credit constraints as a barrier to rentals. We now measure empirically the productivity costs of these delays. To do it, we requires high frequency data which is unavailable to us from our survey data. We instead exploit detailed data from ICRISAT on profitability and value added per acre of about 6200 plots throughout the year in 18 villages in India during 2009-2014. We define an optimal planting time as the date that maximizes the profits per acre in a given village year. Then, we define the cost of the delay as the difference in average productivity or profitability (depending on the variable of interest) as we move away from the optimal planting date.

Formally, we estimate

$$Y_{i,year} = \beta_0 + \beta_1^{+}(\text{Planting Date-Optimal})_{>0} + \beta_1^{-}(\text{Planting Date-Optimal})_{\leq 0} + \alpha X_{i,year} + \epsilon_{i,year}$$

where X are controls for plot characteristics, farmer, village and time fixed effects. Our estimates for productivity costs (measured in value added per acre) are reported in Table 3. They indicate that each additional date away from the profit maximizing date entails about a 8.5% productivity cost per day relative to average productivity within a 5-day window. Standard errors are clustered at the village level.

Table 3: Productivity Costs of Delays Relative to Optimal Planting Time

	Cost per day, Productivity				
	Whole Sample	5-day around optimal		10-day around optimal	
		Before	After	Before	After
β_1	-41.97	391.1**	-215.7	1,166***	-931.1***
	(26.33)	(140.2)	(146.9)	(338.7)	(298.5)
Observations	6,034	1,461	1,882	1,010	1,221
R-squared	0.408	0.625	0.584	0.706	0.659
Mean of Productivity	10228	11425	10694	11921	10998

With these empirical findings at hand, we now present a structural model where different market arrangements can be evaluated in terms of their productivity implications, distributional effects for delays (and its costs), as well as profitability for market providers.

3 A frictional model of capital rental services in space

3.1 Environment

Consider an economy populated by F farmers that use capital services for production. Capital services are available for rent in frictional rental markets. Farmers differ in the equipment-hours demanded from the rental market. A fraction s of them are "large scale" farmers and demand k_s hours, while the remaining $(1-s)$ fraction are "small scale" farmers, and demand k_{s^-} hours. Farmer's productivity entail two components: the first one is realized before any rental service request (ex-ante); and the second one is realized after the service commitment (ex-post). Differences in ex-ante productivity drive differences in production scale (and therefore land-holdings and equipment-hours demand). Differences in ex-post productivity are related to a purely random shock (e.g. weather) as well as to any delay in rental service provision. These delays induce revenue costs to farmers and are determined in equilibrium as a function of the density of service requests and the service capacity per period. Finally, farmers differ in their locations and therefore the travelling time (and cost) of service.

There are H rental service providers that operate different technologies. Each provider can serve up to o orders a day, using up to $\bar{k}_j$ hours a day.[4] They also vary in their technology for service provision. A fraction h of providers use a first-come-first-served (fcfs) technology, while the remaining fraction $1-h$ has access to a selection technology that allows them to prioritize certain type of service requests(mkt).[5] We assume no systematic differences in providers' geographical location, i.e. the expected travel time for service provision is the same for both providers.

Denote the ratio of farmers to service providers, $f = \frac{F}{H}$, and focus on the case where the market is large, i.e. $F, H \to \infty$ and neither side is infinitely larger than the other, $f \in (0, \infty)$. Providers post prices r_{ij} and a selection criteria (with commitment) simultaneously at the beginning of each period. Geographical considerations for service provision are included into the opportunity cost of moving equipment from a provider to the plot. Then farmers decide whether and which provider to approach generating queues for each available provider. Finally, providers decide which orders to serve given the selection criteria and farming production takes place. Given the large number of providers and farmers we focus on a symmetric mixed-strategy equilibrium where ex ante identical providers and farmers use the same strategy and farmers randomize over the set of preferable providers. The key assumption is that agents find it difficult to coordinate their decisions in a large market.

A type i-farmer's strategy is a vector of probabilities $P_i \equiv (p_{ih}, ...; p_{ih^-}, ...)$ where p_{ij} is the probability of applying to each type j-provider. A type-j provider's strategy consists of rental rates per hour serviced, r_{ij} and a selection rule $\chi_j \in [0, 1]$ for the market provider.

[4]If we assume differences in service capacity between providers there will be an additional tradeoff between capacity and speed of service.

[5]Albeit h and H are assumed exogenous, both of them can be easily endogenized with a costly set up of providers and an associated free-entry condition.

The selection rule applies only when a provider receives requests from both type of farmers, in which case the provider prefers the large scale farm if $\chi_j = 1$, prefers a small large farm if $\chi_j = 0$, and it is indifferent between them for $\chi_j \in (0, 1)$. When the provider receives requests from a single type of farmer type, he randomly selects one farmer for service. Conditional on requesting a service from the fcfs provider, all farmers face the same probability of being first in line.

3.2 Queue lengths as strategies

Each farmer maximizes expected profits from farming trading off the probability of obtaining a rental service and the cost of such a service. The characteristics of their demand (or ex-ante productivity) and geographical locations are as in Assumption 1.

Assumption 1: *equipment-hours demanded by small and large scale farmers satisfy* $k_s > k_{s^-}$.[6] *The expected travel time to servicing small-scale farmers is weakly higher than that for large-scale farmers,* $E(d_s) \leq E(d_{s^-})$.

A convenient object for analysis is the *queue length* which is the expected number of farmers requesting a service from a given provider. From a theory standpoint, when the number of providers and firms grow large, the probability of requesting a services to a given provider approaches zero and it is inconvenient to work with. From an empirical standpoint, queues are directly observable from our rental requests data while probabilities are not.

Let q_{ij} be the queue length of type i farmers that apply to a type j firm, where $i \in s, s^-$ and $j \in fcfs, mkt$. Then, $q_{sj} = sFp_{sj}$ and $q_{s^-j} = (1-s)Fp_{s^-j}$. We will refer to $Q_i \equiv \left(q_{i,fcfs}, ...; q_{i,mkt}, ...\right)$ as farmer i strategy. The probabilities of approaching different providers for a single farmer should add up to one, which leads to

$$H\left(hq_{s,fcfs} + (1-h)q_{s,mkt}\right) = Fs \tag{1}$$

$$H\left(hq_{s^-,fcfs} + (1-h)q_{s^-,mkt}\right) = F(1-s) \tag{2}$$

The probability of being served by a given provider Δ_{ij} depends on the scale of the farmer, i, through the size of his order, k_i and the type of provider, j through the service

[6] These differences in demand are induced by differences in ex-ante productivity as we show in section 3.3.

capacity $\bar{k}_j$.

Assumption 2: *Service demand satisfies,*

$$o(k_{s^-} + E(d_{s^-})) <= \bar{k}_j \qquad \text{and } (o+1)(k_{s^-} + E(d_{s^-})) > \bar{k}_j$$

$$(o-1)(k_s + E(d_s)) <= \bar{k}_j \qquad \text{and } (o-1)(k_s + E(d_s)) + k_{s^-} + E(d_{s^-}) > \bar{k}_j$$

$$(o-1)(k_{s^-} + E(d_{s^-})) + k_s + E(d_s) <= \bar{k}_j \qquad \text{and } (o-1)(k_{s^-} + E(d_{s^-})) + 2(k_s + E(d_s)) > \bar{k}_j$$

In other words, for tractability we assume that the service capacity $\bar{k}_j$, is enough to serve at most "o" orders; and that if the provider serves only large-scale orders, it can only serve "o-1" orders. Finally, we assume enough capacity to serve $o-1$ small scale requests and 1 large scale request.

A farmer of scale i that requests service from provider j gets served with probability Δ_{ij}, which depends on the provider's selection criteria and the number of orders it can potentially serve within each period o. We assume the empirically relevant capacity of $o = 3$.[7] The service probability is the sum across all possible number of orders served, $\bar{o} \in \{1, 2, 3\}$ of the probabilities of servicing $\bar{o}$ type i farmers times the probability that a certain farmer of type i is chosen, $\tilde{\Delta}_{ij}(\bar{o})$. These probabilities are characterized in detail in Appendix A. Here we broadly sketch their form.

Consider a farmer of type i that requests a service from a provider j given that other farmers request services with probability $p_{i,j}$. The probability that a given farmer is being served given that the provider chooses one farmer of type i and he is queuing with this provider is $\tilde{\Delta}_{ij}(1)$, i.e.

$$\tilde{\Delta}_{ij}(1) = \sum_{n=0}^{f_i-1} \binom{f_i - 1}{n} p_{ij}^n (1 - p_{ij})^{f_i-1-n} \frac{1}{n+1}.$$

where f_i is the number of farmers of type i searching for a provider, $f_s = sF$ and $f_{s^-} = (1-s)F$ and $\binom{f_i-1}{n} = \frac{f_i-1!}{n!(f_i-1-n)!}$. Hence,

$$\tilde{\Delta}_{ij}(1) = \frac{1 - (1 - p_{ij})^{f_i}}{f_i p_{ij}}$$

[7] This is consistent with the median number of orders served within a day in our admin data.

As the number of agents in the economy gets large, and using the definition of queue lengths above, the service probability simplifies to

$$\tilde{\Delta}_{ij}(1) = \frac{1 - e^{-q_{ij}}}{q_{ij}}$$

That is, a given farmer of type i is served if at least one farmer of type i has requested a service, which occurs with probability $1 - e^{-q_{ij}}$, divided by the number of requests of a given type, q_{ij}.

Next, consider the probability of a given farmer being served when the provider serves $\bar{o} = 2$ orders of type i, $\tilde{\Delta}_{ij}(2)$. Similar computations to those above yield a service probability as follows

$$\tilde{\Delta}_{ij}(2) = 2(\frac{1 - e^{-q_{ij}}}{q_{ij}} - e^{-q_{ij}})$$

Finally, consider the probability of a given farmer being served when the provider serves $\bar{o} = 3$ orders of type i, $\tilde{\Delta}_{ij}(3)$, which follows

$$\tilde{\Delta}_{ij}(3) = 3(\frac{1 - e^{-q_{ij}}}{q_{ij}} - e^{-q_{ij}} - \frac{1}{2}e^{-q_{ij}}q_{ij})$$

In what follows we characterize the probability that a provider of type j services a farmer of type i given that a farmer of scale i is standing in the queue. We characterize these probabilities assuming that at least 3 farmers of either scale have requested service to this provider.[8] Further, we discuss the probabilities of serving exactly one farmer of a given scale by each provider; the remaining probabilities are relegated to Appendix A.

First-come-first-served. The fcfs provider only considers feasibility and the position in the queue. Let the probability of serving $\bar{o}$ farmer of type i be $\phi_{i,fcfs}(\bar{o})$. Given the queue lengths at this provider, there are ${}^{q_s+q_{s-}}P_o = \frac{q_s+q_{s-}!}{(q_s+q_{s-}-o)!}$ possible permutations for the o-tuple, (the provider identifier has been dropped for notational convenience). Under Assumption 1, a fcfs provider serves a single large-scale farmer if one of the large-scale farmers are among the first three positions in the queue and at least one has applied. Let this probability be

[8]When solving for the equilibrium we will show that this is not a binding constraint.

$\hat{\phi}_{s,fcfs}(1) \equiv 3q_s \frac{q_{s^-}P_2}{q_s+q_{s^-}P_3}$

$$\phi_{s,fcfs}(1) = \psi_{s,fcfs}(1)\hat{\phi}_{s,fcfs}(1).$$ [9]

where $\psi_{s,fcfs}(1) = (1 - e^{-q_{s^-,fcfs}} - q_{s^-,fcfs}e^{-q_{s^-,fcfs}})$ is the probability of having at least three orders in the queue of which at least two are of type s^-, when a single farmer of type s has requested service.

A fcfs provider serves a single small-scale farmer if there is one of them in the first o positions of the queue. This probability is defined analogously to its counterpart for large scale orders, exchanging indexes,

$$\phi_{s^-,fcfs}(1) = \psi_{s^-,fcfs}(1)\hat{\phi}_{s^-,fcfs}(1)$$

The general form for the probability of service is,

$$\Delta_{i,fcfs} = \sum_{\bar{o}=1}^{3} \phi_{i,fcfs}(\bar{o})\tilde{\Delta}_{i,fcfs}(\bar{o}) \tag{3}$$

The main difference in the probability of service for large and small scale orders relies on the queue lengths. If the queue lengths are identical, then a first-come-first-served provider serves both types of farmers with the same probability, $\sum_{\bar{o}=2}^{3} \phi_{s^-,fcfs}(\bar{o}) = \sum_{\bar{o}=2}^{3} \phi_{s,fcfs}(\bar{o})$.

Market. The market provider has a technology that allows him to prioritize farmers of either type. The probability of interest is the probability that exactly $\bar{o}$ farmers of type i are served conditional on the farmer under consideration having applied and at least 3 farmers of either type requesting service to the provider.

Conditional on a large farmer having applied, a single large-scale farmer is served by a market provider if the provider does not prioritize large scale farmers and there is one large-scale order among the first o available positions, which happens with probability $(1 - \chi)\tilde{\phi}_{s,mkt}(1) = (1 - \chi)\phi_{s,fcfs}(1)$; or if the provider prioritizes large scale farmers and no other large-scale farmer requested service, $\chi\psi_{s,mkt}(1) = \chi e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}})$.

[9]Note that $\hat{\phi}_{s,fcfs}(i)$ are not the expected probabilities, but rather the probability conditional on the observed queue length. We can numerically show that when $F, H \to \infty$ these two are arbitrarily close.

These service probabilities add up to,

$$\phi_{s,mkt}(1) = \chi\psi_{s,mkt}(1) + (1-\chi)\tilde{\phi}_{s,mkt}(1)$$

Analogous arguments can be used to describe the probabilities of service of small scale farmers. A single small-scale farmer is always served by a market provider (conditional on a request) if it prioritizes high-scale requests and at least two large scale farmers have requested service, which occurs with probability $\chi\psi_{s^-,mkt}(1) = \chi(1-e^{-q_{s,mkt}} - q_{s,mkt}e^{-q_{s,mkt}})$; or if the provider does not prioritize high-scale requests and there is a single small-scale order among the first three orders in the queue, $(1-\chi)\tilde{\phi}_{s^-,mkt}(1)$, where $\tilde{\phi}_{s^-,mkt}(1) = \phi_{s^-,fcfs}(1)$. The reason for always serving a small scale order even when prioritizing large scale is that capacity constraints allow the provider to served at most $o-1$ orders leaving always and idle slot.

$$\phi_{s^-,mkt}(1) = \chi\psi_{s^-,mkt}(1) + (1-\chi)\tilde{\phi}_{s^-,mkt}(1).$$

In sum, the probability of service for a market provider follows

$$\Delta_{i,mkt} = \sum_{\bar{o}=1}^{3} \phi_{i,mkt}(\bar{o})\tilde{\Delta}_{i,mkt}(\bar{o}) \tag{4}$$

Following equations 3 and 4, the probability of being served for a given type i (weakly) declines in the queue length of the other type of farmers. In the first-come-first-served provider the result is straightforward. For the market provider, the decline in the probability of service is strict for the small scale farmers and independent of the queue length of small-scale orders when the provider prioritizes large-scale orders.

3.3 Market cost of provision.

We follow Burdett et al. (2001) and describe a farmer's decision as a function of the market price it would get for the rental service, r_{ij}, which in turn determines its expected "market" profits, U_i. The agents take this value as given when the number of agents in the economy is large, $F, H \to \infty$. Let $q_j \equiv q_{sj}, q_{s-j}$ is the queue of large and small scale farmers queuing at provider j. Each farmer minimizes costs given these "market" profits

$$C_i(r_{ij}, q_j) = \min_{j', k_i} C_i(k_i, r_{ij'}, q_{j'}),$$

subject to

$$\Delta_{ij}\pi_{ij}(k_i, r_{ij}, q_j) \geq U_i,$$

and the technology for production

$$z_i k_i^\alpha \geq y_i,$$

where the cost of production is $C_i(k_i, r_{ij'}, q_{j'}) \equiv k_i r_{ij'}$ and z_i is a type-specific ex-ante productivity. Given the optimal capital demand,

$$k_i = \left(\frac{z_i \alpha}{r_{ij}}\right)^{\frac{1}{1-\alpha}}$$

the expected cost of service is $C_{ij} = r_{ij}^{\frac{-\alpha}{(1-\alpha)}}(z_i\alpha)^{\frac{1}{1-\alpha}}$, a decreasing function of the rental rate r and an increasing function of the ex-ante productivity, z. A large-scale farmer is assumed to have higher ex-ante productivity than a small scale farmer, $z_s > z_{s^-}$ and therefore larger demand for services consistently with Assumption 1.

The profits of a farmer that receives services from provider j are

$$\pi_{ij}(r_j) = \left(\frac{z\alpha}{r^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1}{\alpha} - 1\right) = C_i\left(\frac{1}{\alpha} - 1\right). \tag{5}$$

This expression summarizes the tradeoff between potentially lower provision cost and higher profits, $\pi_i(r_{ij})$ and a lower probability of service, Δ_{ij}. Given the shape of the probability function, there exist a unique queue length $q(r_{ij}, U_i)$ that satisfies the problem of the farmer.

A type i farmer requests a service from a type j firm with positive probability if the expected profits are larger than or equal than U_i. The strict inequality cannot hold because then a type i farmer would apply to that provider with probability 1, yielding $q_{ij} \to \infty$ as their number grows large. Then, $\Delta_{ij} \to 0$ which contradicts, $\Delta_{ij}\pi_{ij}(r_j) > \tilde{U}_i$. The farmers' strategy is

$$q_{ij} \in (0, \infty) \qquad \text{if} \qquad \Delta_{ij}\pi_i(r_j) = \tilde{U}_i \tag{6}$$

$$q_{ij} = 0 \qquad \text{if} \qquad \Delta_{ij}\pi_i(r_j) < \tilde{U}_i \tag{7}$$

The farmer decides his queueing strategy and the size of the equipment service order k_i as a function of his expected productivity, z. Farm productivity entails an exogenous and an endogenous component. The exogenous component is heterogenous across farmers, fixed and known at the beginning of the season, z. The endogenous component of productivity is associated to losses due to mistakes in the timing of agricultural activities, e.g. delays in the provision of equipment. To ease the exposition, we summarize these activities by a "land preparation" date.

The actual land preparation date in a season is a random draw, θ, from a known distribution $G(\theta, q_{ij})$ with mean $\bar{\theta}_z$ and that depends on the equilibrium queues for different farmer types across service providers. These queue lengths determine the probability of service. At the beginning of each season, the farmer chooses where to queue for equipment, which is the same as choosing his optimal expected land preparation date θ_z. If the realization of the preparation date differs from the optimal, the farmer faces a productivity cost proportional to the delay relative to the optimal date. In particular,

$$\bar{z} = z + \eta(\theta - \theta_z)I_{\theta_z \geq \theta}$$

where η is the cost associated to delays in land preparation. Because the draw of the service provision is idiosyncratic, there is no aggregate uncertainty in the economy and factor prices are time independent.

3.4 Service Providers

A service provider with capacity j maximizes expected returns. The stocks of machine-hours available to a provider are exogenously given at $\bar{k}_j$. For simplicity, we assume no depreciation or capital accumulation and no maintenance costs. The cost of servicing a farmer depends on its location relative to the provider. The location of each plot is indexed by d_i and $\mathbf{d}_{\hat{q}_t}$ is a vector collecting the locations of the orders completed within a period, $\hat{q}_t$.

Consider first the problem of a first-come-first-served provider. His value is the expected return from servicing at most o orders within each period. While the provider chooses the cost of service r_{fcfs}, it takes the orders in the queue as given, i.e. in other words, each provider is small. Consistently with the service probabilities, we assume that at least three farmers stand in line at each provider. The provider can serve three orders of the small scale, or two orders of one type and 1 order of another. The unconditional probability of service differ from the previously defined $\phi_{i,fcfs}(\bar{o})$, i.e. the probability of service conditional on a farmer of type i having placed a service request.[10]

The value for a first-come-first-served provider is

$$\begin{aligned} V(\mathbf{q_{fcfs,t}}) \;&=\; \max_{r_{i,fcfs}} \sum_{i=s,s^-} \Phi_{i,fcfs}(2)\left[2\left((r_{i,fcfs}-w)k_i - wE(d_i)\right) + (r_{i',fcfs}-w)k_{i'} - wE(d_{i'})\right] + \\ &\quad \Phi_{s^-,fcfs}(3)3((r_{s^-,fcfs}-w)k_{s^-} - wE(d_{s^-})) + \\ &\quad \beta E\left[V(\mathbf{q}_{fcfs,t+1})\right]. \end{aligned} \tag{8}$$

subject to farmers' strategies, equation 6, and feasibility

$$\sum_{i\in\hat{q}_{fcfs}} k_s(i) + E(d_s(i)) \le \bar{k}_{fcfs} \qquad q_{t+1} = x_{t+1},$$

where $\hat{q}$ correspond to the first three orders in the current queue q_t, and x_{t+1} are draws from the population of farmers that determine the queue length. The queue length is reset each period and therefore the problem is effectively static. The cost of servicing a plot depends on the travelling time, as accounted by the expected travel time $E(d_{s(i)})$ and therefore the foregone services that could have been provided, and the opportunity cost of the driver (with wage w). As in Shi (2002), the provider takes the functional relationship 6 to solve his problem. Given U_i, he chooses the queue lengths by picking the cost of service and its service strategy.

Consider now the recursive problem of a market provider. In addition to choosing the cost

[10]The unconditional probabilities are $\Phi_{i,fcfs}(1) = \Phi_{i',fcfs}(2) = (1-e^{-q_{i,j}})(1-e^{-q_{i',j}} - q_{i',j}e^{-q_{i',j}})\frac{3q_{i,j}q_{i',j}(q_{i',j}-1)}{q_{i,j}+q_{i',j}P_3}$ and $\Phi_{s^-,fcfs}(3) = \left(1-e^{-q_{s^-,j}}(1+q_{s^-,j}+\frac{1}{2}q^2_{s^-,j})\right)\frac{q_{s^-,j}P_3}{q_{s,j}+q_{s^-,j}P_3}$.

of provision, r_{mkt} the provider chooses its optimal selection criteria χ. We can characterize the problem of a market provider as

$$\begin{aligned} V(\mathbf{q_{mkt,t}}) &= \max_{\hat{q}, r_{i,mkt}} \sum_{i=s,s^-} \Phi_{i,mkt}(2) \left[2\left((r_{i,mkt} - w)k_i - wE(d_i)\right) + (r_{i',mkt} - w)k_{i'} - wE(d_{i'})\right] + \\ &\quad \Phi_{s^-,mkt}(3)3((r_{s^-,mkt} - w)k_{s^-} - wE(d_{s^-})) + \\ &\quad \beta E\left[V(\mathbf{q}_{mkt,t+1})\right] \end{aligned} \tag{9}$$

subject to farmers' strategies 6 and feasibility

$$\sum_{i \in \hat{q}_{mkt}} k_s(i) + E(d_s(i)) \leq \bar{k}_{mkt} \qquad q_{t+1} = x_{t+1},$$

where $\hat{q}$ are the orders within the queue q_t that maximize the value to the provider.[11] Again, the provider takes the functional relationship 6 as given when solving his problem.

Before characterizing the equilibrium, it is important to highlight that the problem of the providers is separable across vacancies o, except through the capacity constraint.

4 Symmetric Equilibrium

Consider the ratio of farmers to hubs as exogenous, $\frac{F}{H}$, as well as the fraction of providers that serve on a first-come-first-served basis, h.

A symmetric equilibrium consists of farmers expected profits U_s, U_{s^-}, provider strategies r_{ij}, χ, and farmer strategies, (q_{ij}) where $i = s, s^-$ and $j = fcfs, mkt$, that satisfy:

1. Given U_s, U_{s^-} and other providers' strategies, each type j provider solves 9 and 8,
2. Observing the providers' decision, each farmer satisfies, 6, and
3. U_s, U_{s^-} through q_{ij} are consistent with feasibility, equations 1 and 2.

[11] The relevant unconditional probabilities are $\Phi_{s,mkt}(2) = \chi(1 - e^{-q_{s,mkt}} - q_{s,mkt}e^{-q_{s,mkt}} - \frac{1}{2}q_{s,mkt}^2 e^{-q_{s,mkt}} e^{-q_{s^-,mkt}}) + (1-\chi)\Phi_{s,fcfs}(2)$, $\Phi_{s^-,mkt}(2) = \chi q_{s,mkt} e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}}) + (1-\chi)\Phi_{s^-,fcfs}(2)$, and $\Phi_{s^-,mkt}(3) = \chi e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}} - \frac{1}{2}q_{s^-,mkt}^2 e^{-q_{s^-,mkt}}) + (1-\chi)\Phi_{s^-,fcfs}(3)$.

One can characterize the queue lengths and the selection strategy of the providers separately from the equilibrium cost of service.

Proposition 1. *In all symmetric equilibria, the selection process is $\chi = 1$ and the expected profits of large-scale farmers is higher than for low-scale farmers, $U_s > U_{s^-}$. The per period profit for a provider that serves both type of farmers, $\bar{V}_j$, satisfies:*

$$\bar{V}_{i,j} = \beta^j_{1,i}(q_s, q_{s^-}, \zeta_{\Delta,q}, \zeta_{\phi,q}, \alpha)w(k_s + E(d_s)) - \beta^j_{2,i}(q_s, q_{s^-}, \zeta_{\Delta,q}, \zeta_{\phi,q}, \alpha)w(k_{s^-} + E(d_{s^-}))$$

where $\beta(1,i)^j, \beta(2,i)^j$ are non-linear functions of the queue lengths, the elasticity of the service probabilities with respect to the length of the queues, ζ, and the share of capital in farming production.

A few characteristics are worth discussing. First, differences in location and the cost of travel explain disparities in the incentives to serve farmers operating different scales. In other words, for two plots located at the same distance to the provider, the marginal cost of service is lower for larger scale farmers. On the farmer side, his expected profit depends on its own productivity and inversely on the rental rate of service. If the queues of large and small-scale farmers with a provider had the same lengths, the provider would not find it profitable to serve small-scale farmers and $\bar{V}_{s^-,j} < 0$. In addition, ceteris paribus, the value of servicing a small-scale farmer, $\bar{V}_{s^-}$, is lower for the market provider than for the fcfs provider. A relatively higher queue of small-scale requests increases the value of serving those farmers as it increases their density in the queue. This also implies that small scale farmers are face higher expected delays ceteris paribus.

In equilibrium, farmers that get served from both providers shall be indifferent between either type of them. If queues were identical across providers, equalizing expected profits to the farmers would imply that the cost of service, r, was the same across farmers of different scales in equilibrium. But this cannot be the case since the marginal cost of service for large-scale farmers is smaller than for small-scale farmers. In addition, because market providers value large-scale orders weakly higher than fcfs providers, the equilibrium shall display longer queues of large-scale farmers to the market provider.

5 Quantitative assessment

In this section, we characterize expected delay, productivity costs, and provider profitability for different dispatching arrangements under equilibrium conditions specified by the theoretical model using transaction data on one CHC/implement combination as an illustration. In particular, we ask whether the FCFS dispatch system changes equipment allocations relative to the market dispatch system both in space and across farmers of different sizes. This characterization is important in assessing the impact of the characteristics of the rental market for market accessibility and overall productivity. We then perform the same analyses on two counterfactual cases. In the first case, FCFS providers are allowed to behave like a market provider, i.e. to choose a dispatching arrangement that maximizes profits while maintaining the overall service capacity unchanged; in the second case, we explore a long-run equilibrium where there are only market providers and the overall service capacity is allowed to adjust so that the industry is earning zero profit.

5.1 Data

We use the platform transaction data of rotavator (6 feet) in CHC Kasaba (Hunsur) from year 2018 in the kharif season (June to October).[12] We focus on rotavator as it is the most commonly rented during this season. Kasaba (Hunsur) has one of the highest numbers of transactions of rotavator (105 transactions observed). For each transaction in the data, we observe the amount of hours requested, the acreage serviced, the total cost of the service, the implement rented, and the village from which the farmer is requesting the service.

5.2 Empirical strategy

There are two blocks to the quantitative assessment that can be solved separately. The first involves solving for the market prices of services and queue lengths using the theoretical model. As discussed in the previous section, there exists multiple equilibria in terms of what type of farmers is served by what type of providers. Without imposing an equilibrium selection mechanism, the theoretical model does not predict which equilibrium will arise. To

[12]Future versions of the draft will include data from other hubs.

determine what is the most likely equilibrium, we resort to the transaction data. One key difference between the theoretical model and the data is that the size of a farmer, measured by the number of hours of service demanded, is not binary but continuous in the data. To bridge the gap, we define a large farmer as one that request 3.5 hours of service or above; the rest are small farmers.[13] What we observe in the platform transaction data is that both types of farmers, large or small, are served by the FCFS providers. The census data show that both are also served by the market providers. As a result, we solve for an in equilibrium in which both types of farmers are served by both types of providers (the "status quo" equilibrium).

The second block involves finding the expected delay and subsequent productivity costs as well as provider profitability under the FCFS and market systems respectively using the market prices and queue lengths of the status quo equilibrium from the first block. In theory, the queue length itself does give us information on expected wait time. However, we recognize the fact that the data we have contain much richer information on farmer heterogeneity that is not reflected by the stylized theoretical model. For example, the size of farmers, as mentioned earlier, is not binary in the data. Further, the theoretical model is not rich enough to shed light on the impact of spatial allocation of farmers should it be correlated with the size of the farmer. As a result, we use a simulation approach to quantify expected delay, productivity costs and provider profitability, taking advantage of the transaction-level information on farmer's demand and location in the data. We first estimate a function that maps a given queue of orders to a net present value for each dispatch system using the equilibrium rental prices and queue lengths as inputs, and use this function to simulate the order of service delivery for samples of queues drawn from the empirical distribution.

5.3 Parameterization

Table 4 shows the list of parameters required for the quantitative assessment and how are they determined. The share of FCFS providers (h), share of large farmers (s), and returns to scale (α) are all endogenously determined from data. The cost of moving a piece of equipment to a location and operating it is parameterized as a linear function of the opportunity cost

[13] 3.5 hours is the median request for farmers that hold 4 acres or more of land.

of time. To measure this opportunity cost (w), we use information on average per-day wages paid to drivers fulfilling orders in a CHC.

To make sure the equilibrium queue lengths are in line with what we observe in the data, we calibrate f, the ratio between the number of farmers and number of equipment by targeting the queue length of small farmers served by the FCFS providers ($q_{s-,fcfs}$) in the platform data.

To discipline the productivity costs of delays, η, we use the estimates presented in Section 2.[14] An important input to assess the cost of delays is the productivity distribution of the farmers that are ordering different service-hours. To discipline the distribution, we use a portion of the transactions' sample that overlaps with the survey data (approximately, 1300 observations) and compute the underlying correlation between productivity and service-hours. We assume that the distribution of productivity is log-normal, $ln(z) \sim \mathcal{N}(\mu, \sigma)$ and fit a parametric distribution via maximum likelihood.

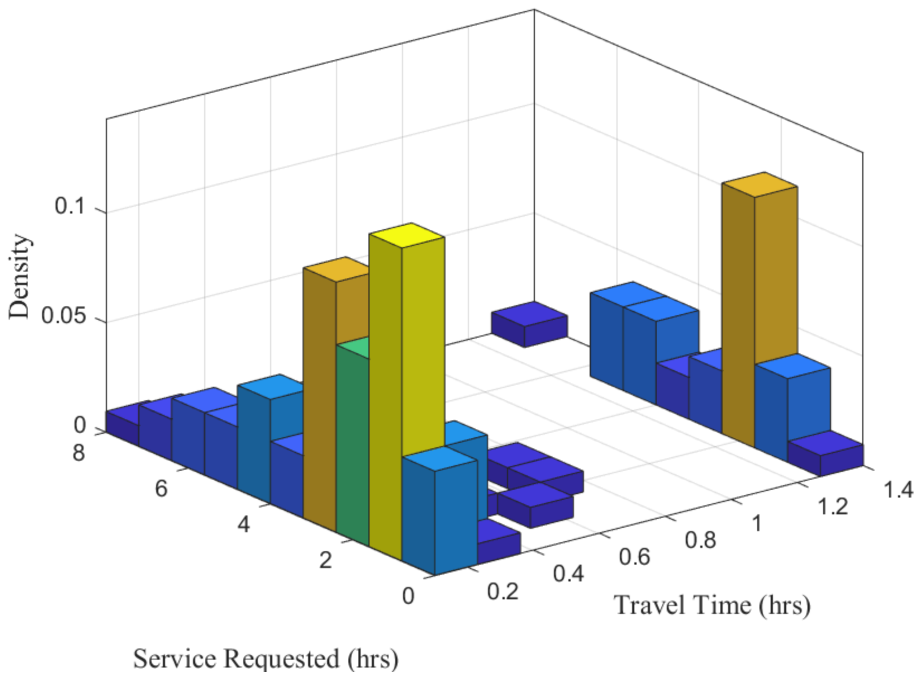
Table 4: Parameterization

Parameter	Description	Value	Source/Moment
Exogenous parameters			
h	Share of FCFS providers	0.75	Census data
s	Share of large farmers	0.1	Survey data
α	Returns to scale	0.6	Survey data
β	Discount factor	0.98	-
w	Travel/op. cost (INR/hr)	75	Platform data
η	Productivity loss/day	3%	ICRISAT sample
μ	Log-normal mean of productivity	2.19	Survey Data
σ	Log-normal s.d. of productivity	0.17	Survey data
Calibrated parameters			
f	No. of farmers/No. of equipment	3.9	$q_{s-,fcfs} = 3.0$

As discussed in the empirical strategy, the platform data contain information on the joint distribution of number of hours of service demanded and the location, measured by its distance to the CHC, of the farmers. Figure 6 depicts this joint distribution for Kasaba (Hunsur). There are two large clusters of farmers, one located near the CHC within a travel

[14] We use the costs in terms of productivity rather than profitability since the estimated costs per day are smaller for productivity than those suggested by profitability.

Figure 6: Order distribution by service requested and travel time



Data from Kasaba (Hunsur) for a Rotavator 6 feet.

distance of 0.4 hours; the other is further away with a travel distance between 1.2 and 1.4 hours. Within each cluster, the distribution of hours demanded is similar, with the majority of farmers demanding 1 to 3 hours. In the simulation exercise, we draw orders directly from this distribution adjusted by the relative measure between the large and small farmers implied from equilibrium queue lengths.

5.4 Status quo equilibrium

We solve for the rental rates and queue lengths under the status quo equilibrium using the theoretical model. In the status quo equilibrium, both types of farmers have access to both types of providers and therefore, each type of farmers is indifferent between being served by a FCFS provider and a market provider (see Appendix B for details). Table 5 summarizes the equilibrium outcome. The rental prices charged by a market provider are lower for both types of farmers compared to a FCFS provider. These differences in rental prices partly explain why we observe longer queue lengths at the market provider for both types of farmers.

Table 5: Status Quo Equilibrium Outcome

Rental Rates per Hour

	FCFS	Market
Small farmer	358	354 (-1%)
Large farmer	373	265 (-30%)

Avg. Queue Length per Equipment

	FCFS	Market
Small farmer	3.0	5.0 (+67%)
Large farmer	0.34	0.53 (+55%)

The percentages in parentheses are comparing to the corresponding metrics under FCFS

5.5 The value of service and the cost of delays

We first estimate the value function for each provider, i.e. a function that maps a queue of orders to their service value. We fix the number of orders each provider with 1 piece of equipment can serve in a day to 3, in line with the theoretical model. As we explain in Appendix **??**, this is a high dimensional problem, and the number of possible combinations of orders to be served within a period (and their continuation values) grows exponentially with the number of orders in the queue and its characteristics (including hours serviced, latitude and longitude). The disparities in delays between the FCFS allocation and the "market" one depend on the capacity of the CHC relative to the hours in the queue, or the degree of market congestion.

The equilibrium queue lengths we solved earlier have direct implications for the distribution of orders in the economy. This distribution is not necessarily identical to the empirical distribution realized in the data. To reconcile the discrepancy, we assume that the observed transactions in the platform are indeed representative of the population distribution of orders within each type of farmers. Across the two types, we adjust the ratio of the overall measure of each type to match the ratio of their respective equilibrium queue lengths.

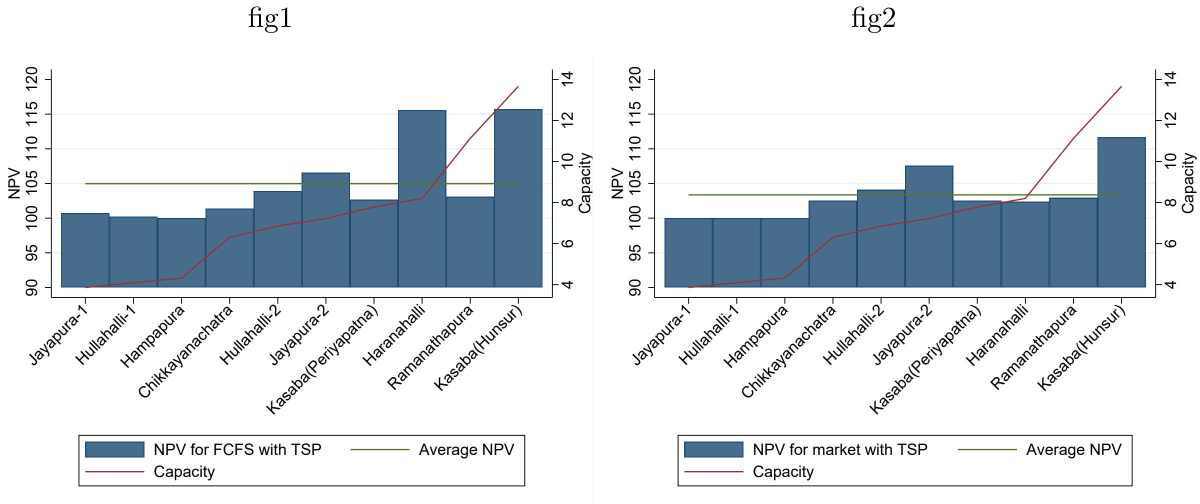
To compute the travel time between two locations $\nu_{i,j}$ we exploit an application programming interface (API) that geolocates locations and CHC's and computes travel time by car (similarly to the predictions in google maps). Rural Karnataka is not always well connected and while geographical distances may seem short, travel times increase rapidly. The travel time to locations is typically bounded above by half an hour from the CHC and the distribution of service requests in the hours space varies substantially across CHC's and across farmers within a CHC.

Given this information we compute the value function that solves the provider problem under each alternative dispatch system. Within each system, we consider two delivery routes. One follows a "hub and spoke" pattern, under which the equipment must return to the CHC between two orders. The other solves a "Travelling Salesman Problem (TSP)" where the implement travels directly from order to order. Under FCFS, this means that the provider follows the route that minimizes the travel time within a given day. Under the market allocation, the choice of route is part of the dynamic problem entailing the choice of which orders to serve given their value. The value of an order in the market allocation depends on the density of orders around them, and the size of the order relative to the CHC serving capacity. In this version of the draft, we only allow the providers to follow the first dispatch system. Since for any queue of orders, the travel time under the TSP system is no longer than that under the hub and spoke pattern. Therefore we can expect an enhancement in value to the providers should we allow them to operate under the TSP system. As an illustration, we include value comparisons for other CHCs (see Fig 12). These values are computed without solving for an equilibrium. In other words, we assume that the markets are already in equilibrium.

To understand the implications of different dispatch systems for delays in equipment rental provision we simulate queues by sampling (with replacement) from the empirical distribution of orders re-weighted by the ratio between equilibrium queue lengths of large and small farmers. Then, we numerically generate the equilibrium distribution of delays for farmers of different characteristics. Details can be found in Appendix C. When assessing the merits of different dispatch arrangements for observed delays and productivity costs, we draw multiple samples from the productivity distribution and make sure that the correlation between the productivity realizations and the service hours is as in the data (benchmark). We also assess outcomes assuming no correlation, and negative correlation between productivity and order sizes.

We subsequently explore the effects of these two alternative dispatch systems for farmers

Figure 7: Value enhancement with TSP(NPV without TSP normalized to 100)



with different service-hours requests. To do so, we use the discretization of the space of service orders and report average waiting times for each bin separately (see Figure 8). We find that waiting time is higher for all orders except for the largest ones under the market allocation.

Last, we compare productivity costs, both at per acre and aggregate level, under the two dispatch systems (see Figure 9). We find that both costs are higher under the market system for all order sizes. As discussed earlier, wait time under the market system is longer than under FCFS with the exception of the largest orders. This feature translates into a higher (unweighted) average productivity cost under the market system for the smaller orders. For the largest orders, the higher costs under the market system are caused by heterogeneity in productivity across hubs. Among farmers who place the largest orders, those who experience longer wait time under the market system tend to locate in locations with higher average productivity, thus experiencing larger productivity costs in absolute value. Despite a shorter (unweighted) average wait time under the market system, the larger losses by these more productive farmers lead to higher average productivity cost both per acre and in aggregation.

5.6 Counterfactual analysis

In the first counterfactual case, we explore a scenario in which the technology that allows the market providers to prioritize large farmers is made available to the FCFS providers. Given

Figure 8: Wait time for heterogenous sizes under alternative dispatch systems

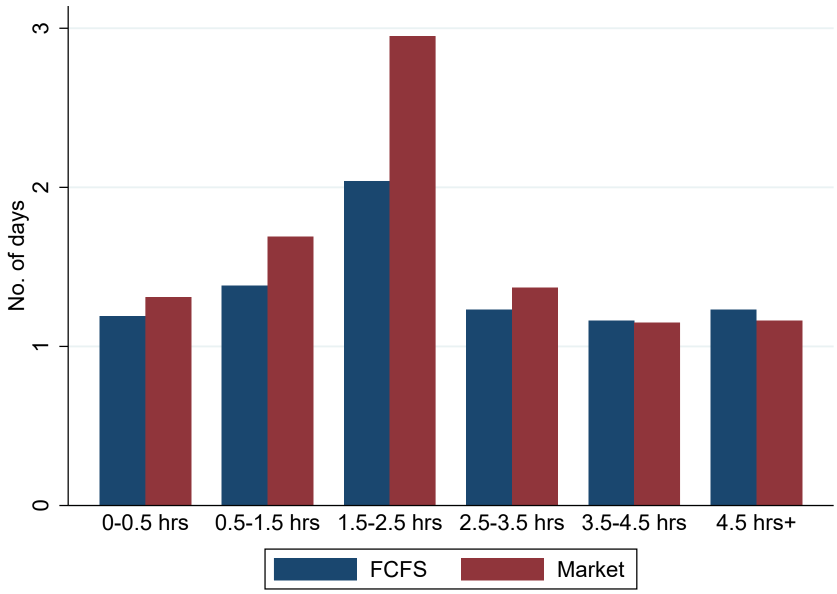
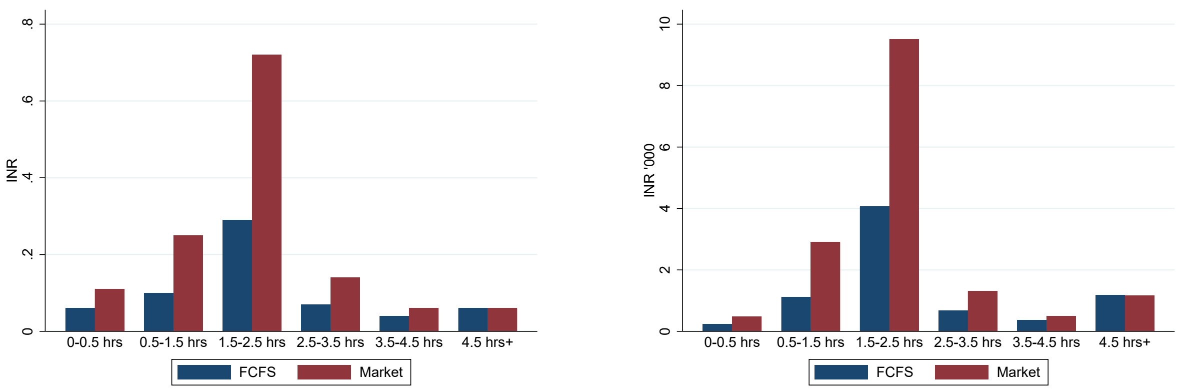


Figure 9: Productivity costs for heterogeneous sizes



Per-acre productivity costs

Aggregated productivity costs

this technology, the FCFS providers are at least as well off as before. As a result, a profit driven FCFS provider would choose to adopt the technology, i.e. $h = 0$. In other words, there is no longer any differentiation between these two types of providers from a farmer's perspective. We keep the rest of the parameters unchanged. We also assume that providers cannot exit the market in the short run. We call this the short-run market equilibrium.

The first two panels of Table 6 compare the equilibrium queue lengths, rental rates and expected per period profit of serving one farmer ($V_{i,mkt}$) under the status quo equilibrium and under the short-run market equilibrium. The rental rates charged to both types of farmers under short-run market equilibrium are significantly lower. This is primarily due to the fact that without any differentiation in services, providers compete heads-on with one another, which significantly diminishes their pricing power. Note that the expected per period profit of serving a small farmer is now negative. This suggests that a profit maximizing provider will not serve small farmers.

In the second counterfactual case, we explore what happens in the long run to the equilibrium in the first one when providers can exit the market. Again assume all providers follow the dispatching arrangement of a market provider. Since the expected per period profit of serving a small farmer will be negative as demonstrated in the first case, providers will choose to price the small farmers out in the absence of government intervention. However, given the small number of large farmers, there will be over capacity in the market which leads to negative profits. In the long run, exits take place until the expected profit increases to zero. Panel 3 of Table 6 shows that after the capacity reduction caused by exits, the capacity per farmer (measured by $\frac{1}{f}$) falls by 88% from 0.26 to 0.03. The dramatic reduction in capacity causes the queue length to increase by over 5 times for the large farmers who continue to be served. As a result, they experience much longer waiting time (see Fig. 10).

The counterfactual analysis appears to suggest that the impact from the Indian government's intervention in setting up an FCFS equipment rental system is mixed. On the one hand, by setting up a system that follows a different dispatching system, small farmers that would not otherwise have access to the market are now served. Further, both large and small farmers benefit from the product differentiation. In fact, the product differentiation significantly increases the farmers' willingness to pay, leading to a higher profit level for both the FCFS and the market providers. On the other hand, once the government stops

Figure 10: Wait time for heterogeneous sizes
under status quo equilibrium and long run market equilibrium

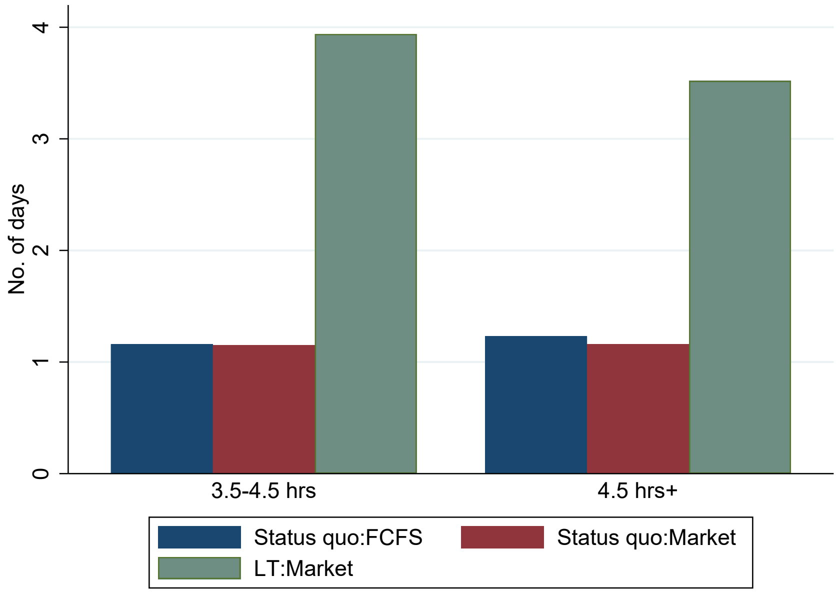


Table 6: Equilibrium Outcome Comparison

	Status quo eqm h=0.8	Short run mkt eqm: FCFS providers turned into mkt providers h=0	Long run mkt eqm: FCFS providers turned into mkt providers with reduced cap. h=0
Capacity per farmer *	0.26	0.26 (+0%)	0.03 (-88%)
$q_{s-,mkt}$	5.0	3.5 (-30%)	-
$q_{s,mkt}$	0.5	0.4 (-26%)	3.2 (+508%)
$r_{s-,mkt}$	354.3	89.3 (-75%)	-
$r_{s,mkt}$	264.9	111.6 (-58%)	93.3 (-65%)
$V_{s-,mkt}$	396.9	-49.6 (NA)	-
$V_{s,mkt}$	789.3	84.2 (-89%)	0 (NA)

* Defined by number of equipment per farmer

regulating the FCFS providers and allows them to become profit driven, they will adopt the dispatching system of the market providers. This will lead to strong competition among all providers and suppress prices to small farmers to the point that it is no longer profitable to serve them. The low level of demand will cause providers to exit in the long run, leading to a significant lower market capacity. Small farmer lose access to the rental services. Large farmers do pay a lower price, but suffer from a longer wait time and greater productivity losses.

6 Conclusion

Rental markets hold considerable promise in expanding mechanization access and increasing productivity in the farming sector. However, the spacial distribution of demand in space and its synchronous nature, as well as the fixed supply capacity, pose interesting trade offs between efficiency and market access. The returns to these rental markets depend crucially on factors such as density, i.e. the proximity of suppliers to farmers, the overall supply capacity, and the ability to optimize travelling equipment time. In this paper, we document and quantify how these factors determine the allocative efficiency and distributional effects of rental markets.

We find that when the government increases service capacity by subsidizing the purchase of equipment from rental service provision, and at the same time imposes a first-come-first-serve dispatch system to allocate services, it induces misallocation in service provision. Indeed, when equipment owners are allowed to behave optimally, by prioritizing larger scale orders (which are the most cost effective) the equilibrium returns to one where smallholder farmers are rationed out of the market.

Future versions of this paper will include a quantification of the full set of equipment suppliers for which we hold data.

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A Probability characterization

FCFS Here we describe the probabilities of servicing more than one order of a given scale.

A fcfs provider services 2 large-scale farmers if there are two or more large-scale orders in the first o positions of the queue and at least two large scale farmers have applied. Let this probability be $\hat{\phi}_{s,fcfs}(2) \equiv 3q_s(q_s-1)\frac{q_s-2+q_{s^-}P_1}{q_s+q_{s^-}P_3}$

$$\phi_{s,fcfs}(2) = \psi_{s,fcfs}(2)\hat{\phi}_{s,fcfs}(2)$$

where $\psi_{s,fcfs}(2) = (1-e^{-q_{s,fcfs}} - e^{-q_{s^-,fcfs}}q_s e^{-q_{s,fcfs}})$ is the probability that there are at least three orders in the queue conditional of a farmer of type s requesting service, of which at least two are of type s (including the one requesting service).[15]

Given feasibility, the fcfs provider never serves 3 large-scale orders, $\phi_{s,fcfs}(3) = 0$.

A fcfs provider services 2 small-scale farmers if at least two small-scale orders in the first o positions of the queue. Let this probability be $\hat{\phi}_{s^-,fcfs}(2) \equiv 3\frac{q_{s^-}(q_{s^-}-1)q_s}{q_s+q_{s^-}P_3}$

$$\phi_{s^-,fcfs}(2) = \psi_{s^-,fcfs}(2)\hat{\phi}_{s^-,fcfs}(2)$$

where $\psi_{s^-,fcfs}(2) = (1-e^{-q_{s,fcfs}})(1-e^{-q_{s^-,fcfs}})$ is the probability that there are at least three orders in the queue conditional of a farmer of type s^- requesting service, of which at least one is of type s and at least two are of type s^- (including the one requesting service).

A fcfs provider services 3 small-scale farmers if there are three small-scale orders in the first o positions of the queue. This probability is defined as $\hat{\phi}_{s^-,fcfs}(3) = \frac{q_{s^-}P_3}{q_s+q_{s^-}P_3}$

$$\phi_{s^-,fcfs}(3) = \psi_{s^-,fcfs}(3)\hat{\phi}_{s^-,fcfs}(3)$$

where $\psi_{s^-,fcfs}(3)$ is the probability of having at least two other small scale requests, i.e. $\psi_{s^-,fcfs}(3) = (1-e^{-q_{s^-,fcfs}} - q_{s^-,fcfs}e^{-q_{s^-,fcfs}})$.

Market Two large-scale farmers are served by a market provider if he does not prioritize large orders and they stand in the first 3 positions, which happens with probability (1 −

[15]This is the probability that at least another large scale and at least one small scale farmer request service, or at least two other large scale farmers request service.

$\chi)\tilde{\phi}_{s,mkt}(2) = (1-\chi)\phi_{s,fcfs}(2)$; or if the provider prioritizes those orders and there is at least one additional large-scale service request, which happens with probability $\chi\psi_{s,mkt}(2) = \chi(1 - e^{-q_{s,mkt}} - e^{-q_{s^-,mkt}} q_s e^{-q_{s,mkt}})$.[16] These service probabilities add up to

$$\phi_{s,mkt}(2) = \chi\psi_{s,mkt}(2) + (1-\chi)\tilde{\phi}_{s,mkt}(2).$$

Feasibility prevents three large-scale orders to be served within the period and therefore, $\phi_{s,mkt}(3) = 0$.

Two small-scale farmers are served by a market provider if it prioritizes high-scale requests and exactly one large-scale farmer requests service and at least another small scale farmer requests service, which occurs with probability $\chi\psi_{s^-,mkt}(2) = \chi q_{s,mkt} e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}})$. Alternatively, two small-scale farmers are served if the provider does not prioritize large-scale orders and there are two small-scale orders among the first three orders in the queue.

$$\phi_{s^-,mkt}(2) = \chi\psi_{s^-,mkt}(2) + (1-\chi)\tilde{\phi}_{s^-,mkt}(2).$$

Three small-scale farmers are served by the market provider if it prioritizes high-scale requests and no large-scale farmer requests service and there are at least three small requests, which occurs with probability $\chi\psi_{s^-,mkt}(3) = \chi e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt} e^{-q_{s^-,mkt}})$, or if it does not prioritize them and there are three small-scale orders among the first three in the queue,

$$\phi_{s^-,mkt}(3) = \chi\psi_{s^-,mkt}(3) + (1-\chi)\tilde{\phi}_{s^-,mkt}(3).$$

B Proofs

1. *If a **provider j attracts only large-scale farmers**, i.e.* $q_{s^-,j} = 0$*, then the expected per period profit of the provider satisfies*

$$\frac{\bar{V}_s}{w(k_s + E(d_s))} = \beta(q_{s,j}, \zeta_{\Delta,q}, \alpha)$$

[16] If there are more large-scale orders the provider still serves two because of its capacity constraints.

where $\bar{V}_i \equiv \big((r_{i,mkt} - w)k_i - wE(d_i)\big)$ *and* $\beta()$ *is a non-linear function of the queue length, the elasticity of the service probability with respect to the length of the queue,* ζ*, and the share of capital in farming production. The expected profit of the farmer satisfies*

$$U_s = \frac{1-\alpha}{\alpha}\tilde{\Delta}_{ij}\frac{(k_s^\alpha z\alpha)^{\frac{1}{1-\alpha}}}{\big((1+\beta)w(k_s + E(d_s))\big)^{\frac{\alpha}{1-\alpha}}}$$

Proof. Let's first describe the problem of the supplier when it only receives large scale orders,

$$\max_{q_{s,j}, r_{s,j}} \psi 2\bar{V}_s$$

subject to

$$\tilde{\Delta}_{s,j}(2)\pi_s(r_{s,j}) \geq U_s$$

$$\sum_{i\in\hat{q}_j} k_s(i) + E(d_s(i)) \leq \bar{k}_j$$

where $\psi = \left(1 - e^{-q_{s,mkt}}(1 + q_{s,mkt} + \frac{1}{2}q_{s,mkt}^2)\right)$ because there are no small-scale orders. Using the definition of profits to the farmers, equation 5, we can replace the cost of capital into the objective function. Replacing the rental price of capital as a function of the expected profits, the provider solves

$$\max_{q_{s,j}} 2\psi\left[(z\alpha)^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\alpha}\frac{\tilde{\Delta}_{sj}(2)}{U_s}\right)^{\frac{1-\alpha}{\alpha}} k_s - w(k_s + E(d_s))\right]$$

Note that the properties of the probabilities ψ and $\tilde{\Delta}$ (decreasing and convex in the queue length) imply that the first order conditions to the problem are necessary and sufficient for an optimum. The optimality condition for the queue length is

$$\frac{\partial\psi}{\partial q}\bar{V}_s + \psi\left(\frac{1-\alpha}{\alpha}\right)\left[\frac{\bar{V} + w(k_s + E(d_s))}{\tilde{\Delta}_{sj}(2)}\right]\frac{\partial\tilde{\Delta}_{sj}(2)}{\partial q} = 0. \tag{10}$$

Let the elasticity of the probability of service with respect to the queue length be $\zeta_{q\Delta}(o) \equiv -\frac{\partial\tilde{\Delta}_{sj}}{\partial q}\frac{q_{sj}}{\tilde{\Delta}_{sj}(o)}$ and let the elasticity of the probability of arrival of o orders to the queue length be $\zeta_{q\psi}(o) \equiv \frac{\partial \psi}{\partial q}\frac{q}{\psi(o)}$. Finally, let $\beta(q_{s,j}, \zeta_{\Delta,q}, \zeta_{\psi,q}, \alpha) \equiv \frac{\frac{1-\alpha}{\alpha}\zeta_{q\Delta}}{\zeta_{q\psi} - \zeta_{q\Delta}\frac{1-\alpha}{\alpha}}$,

$$\bar{V}_s = \beta(q_{s,j}, \zeta_{\Delta,q}, \zeta_{\psi,q}, \alpha) w(k_s + E(d_s)) \tag{11}$$

which proves the result. For the value to be positive we require $\beta > 0$. This is the same as assuming a large enough span of control because β has an asymptote at $\frac{\alpha}{1-\alpha}$ where $\lim^{-}_{\frac{\zeta_{q\Delta}}{\zeta_{q\psi}} \to \frac{\alpha}{1-\alpha}} \beta = +\infty$ and $\lim^{+}_{\frac{\zeta_{q\Delta}}{\zeta_{q\psi}} \to \frac{\alpha}{1-\alpha}} \beta = -\infty$ and $\lim_{q \to \infty} \beta = -1$.

By definition,

$$\bar{V}_s = (z\alpha)^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{\alpha} \frac{\tilde{\Delta}_{ij}}{U_s} \right)^{\frac{1-\alpha}{\alpha}} k_s - w(k_s + E(d_s))$$

Replacing $\bar{V}_s$ by its optimum value we can solve for the expected profits of the farmer,

$$U_s = \frac{1-\alpha}{\alpha} \tilde{\Delta}_{ij} \frac{(k_s^{\alpha} z\alpha)^{\frac{1}{1-\alpha}}}{\big((1+\beta) w(k_s + E(d_s))\big)^{\frac{\alpha}{1-\alpha}}}$$

□

2. *If a* ***provider j attracts only small-scale farmers****, i.e.* $q_{s,j} = 0$*, then the expected per period profit of the provider satisfies*

$$\frac{\bar{V}_{s^-}}{w(k_s + E(d_s))} = \beta(q_{s^-,j}, \zeta_{\Delta,q}, \alpha)$$

where $\bar{V}_i \equiv \big((r_{i,mkt} - w)k_i - wE(d_i)\big)$ *and* $\beta()$ *is a non-linear function of the queue length, the elasticity of the service probability with respect to the length of the queue,* ζ*, and the share of capital in farming production. The expected profit of the farmer satisfies*

$$U_{s^-} = \frac{1-\alpha}{\alpha} \tilde{\Delta}_{s^-j} \frac{\left(k_{s^-}^{\alpha} z\alpha\right)^{\frac{1}{1-\alpha}}}{\big((1+\beta) w(k_{s^-} + E(d_{s^-}))\big)^{\frac{\alpha}{1-\alpha}}}$$

Therefore, it is analogous to the expected value when only large scale are served. Also,

3. *If a* ***provider j attracts large-scale and small-scale farmers****, i.e.* $q_{s^-,j}, q_{s,j} > 0$*, then the expected per period profit of the provider, is a convex combination of the value*

of each service.

$$\bar{V}_i = \beta_{1,i}^{fcfs}(q_{s,j}, q_{s^-,j}, \zeta_{\Delta,q}, \alpha) w(k_s + E(d_s)) - \beta_{2,i}^{fcfs}(q_{s,j}, q_{s^-,j}, \zeta_{\Delta,q}, \alpha) w(k_{s^-} + E(d_{s^-}))$$

where $\beta(1,i), \beta(2,i)$ are non-linear functions of the queue lengths (for both type of farmers), the elasticity of the service probability with respect to the length of the queues, ζ, and the share of capital in farming production.

Proof. Using the definition of the expected profits, we can replace them in the expected per period profit of the provider to obtain

$$\begin{aligned}\max_{q_{s,j}, q_{s^-,j}} \sum_{i=s,s^-} \Phi_{i,j}(2)[(z\alpha)^{\frac{1}{\alpha}} \left(2\frac{1-\alpha}{\alpha}\frac{\Delta_{ij}}{U_i}\right)^{\frac{1-\alpha}{\alpha}} k_i - 2w(k_i + E(d_i)) \\ +(z\alpha)^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{\alpha}\frac{\Delta_{i'j}}{U_{i'}}\right)^{\frac{1-\alpha}{\alpha}} k_{i'} - w(k_{i'} + E(d_{i'}))] \\ +\Phi_{s^-,j}(3)3\left[(z\alpha)^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{\alpha}\frac{\Delta_{s^-j}}{U_{s^-}}\right)^{\frac{1-\alpha}{\alpha}} k_{s^-} - w(k_{s^-} + E(d_{s^-}))\right]\end{aligned}$$

The optimality condition with respect to the queue length of large-scale and small-scale farmers are

$$\begin{aligned}\sum_{i=s,s^-}\left(\frac{\partial \Phi_{i,j}(2)}{\partial q_{s,j}}\left[2\bar{V}_i + \bar{V}_{i'}\right]\right) + 3\frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s,j}}\bar{V}_{s^-} + \\ \sum_{i=s,s^-}\left(\Phi_{i,j}(2)2\frac{\partial \bar{V}_i}{\partial q_{s,j}} + \Phi_{i',j}(1)\frac{\partial \bar{V}_{i'}}{\partial q_{s,j}}\right) + \Phi_{s^-,j}(3)3\frac{\partial \bar{V}_{s^-}}{\partial q_{s,j}} = 0,\end{aligned} \tag{12}$$

$$\begin{aligned}\sum_{i=s,s^-}\left(\frac{\partial \Phi_{i,j}(2)}{\partial q_{s^-,j}}\left[2\bar{V}_i + \bar{V}_{i'}\right]\right) + 3\frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s^-,j}}\bar{V}_{s^-} + \\ \sum_{i=s,s^-}\left(\Phi_{i,j}(2)2\frac{\partial \bar{V}_i}{\partial q_{s^-,j}} + \Phi_{i',j}(1)\frac{\partial \bar{V}_{i'}}{\partial q_{s^-,j}}\right) + \Phi_{s^-,j}(3)3\frac{\partial \bar{V}_{s^-}}{\partial q_{s^-,j}} = 0,\end{aligned} \tag{13}$$

where

$$\frac{\partial \bar{V}_i}{\partial q_{i,j}} = \left(\frac{1-\alpha}{\alpha}\right)\left[\frac{\bar{V}_i + w(k_i + E(d_i))}{\Delta_{ij}}\right]\left(\frac{\partial \Delta_{i,j}}{\partial q_{ij}}\right).$$

This envelope condition is particularly important. Define the elasticity of variable x as $\zeta_x \equiv \frac{\partial x}{\partial q}\frac{q}{x}$, then,

$$\zeta_{\bar{V}_i} = \frac{1-\alpha}{\alpha}\zeta_{\Delta ij}(\bar{o}) + \frac{w(k_i + E(d_i))}{\bar{V}_i}\zeta_{\Delta ij}(\bar{o})$$

If the value of serving an order of scale i is independent of the composition of the $\bar{o}$-ad being served, the elasticity of the value of service to the queue length is a linear function of the elasticity of the probability of service Δ, to the queue length.[17] Then the value of different elasticities for different scales of orders are proportional to each other.

Equations 12 and 13 form a system of linear equations that can be solved for the two unknowns $\bar{V}_{s^-}, \bar{V}_s$ as a function of the queue lengths.[18]

$$\Gamma \begin{bmatrix} \bar{V}_s \\ \bar{V}_{s^-} \end{bmatrix} = a$$

where $\Gamma = \begin{bmatrix} \Gamma_1 \Gamma_2 \\ \Gamma_3 \Gamma_4 \end{bmatrix}$, and

$$\Gamma_1 = \frac{\partial \Phi_{s,j}(2)}{\partial q_{s,j}} 2 + \frac{\partial \Phi_{s^-,j}(2)}{\partial q_{s,j}} - \zeta_{\Delta_s q_s}(2\frac{\Phi_{s,j}(2)}{q_{s,j}} + \frac{\Phi_{s,j}(1)}{q_{s,j}})\left(\frac{1-\alpha}{\alpha}\right)$$

$$\begin{aligned}\Gamma_2 \quad = \quad & 2\frac{\partial \Phi_{s^-,j}(2)}{\partial q_{s,j}} + \frac{\partial \Phi_{s^-,j}(1)}{\partial q_{s,j}} + 3\frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s,j}} - \\ & \zeta_{\Delta_{s^-} q_s}(\frac{\Phi_{s^-,j}(1)}{q_{s,j}} + 2\frac{\Phi_{s^-,j}(2)}{q_{s,j}} + 3\frac{\Phi_{s^-,j}(3)}{q_{s,j}})\left(\frac{1-\alpha}{\alpha}\right)\end{aligned}$$

[17] In this case $\frac{w(k_i+E(d_i))}{V_i}$ is a constant.

[18] Note that equation 12 reduces to 10 when there are no small-scale orders.

$$\Gamma_3 = 2\frac{\partial \Phi_{s,j}(2)}{\partial q_{s^-,j}} + \frac{\partial \Phi_{s,j}(1)}{\partial q_{s^-,j}} - \zeta_{\Delta_s q_{s-}}(\frac{\Phi_{s,j}(1)}{q_{s^-,j}} + 2\frac{\Phi_{s,j}(2)}{q_{s^-,j}})\left(\frac{1-\alpha}{\alpha}\right)$$

$$\begin{aligned}\Gamma_4 &= 2\frac{\partial \Phi_{s^-,j}(2)}{\partial q_{s^-,j}} + \frac{\partial \Phi_{s^-,j}(1)}{\partial q_{s^-,j}} + 3\frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s^-,j}} - \\ &\quad \zeta_{\Delta_{s-} q_{s-}}(\frac{\Phi_{s^-,j}(1)}{q_{s^-,j}} + 2\frac{\Phi_{s^-,j}(2)}{q_{s^-,j}} + 3\frac{\Phi_{s^-,j}(3)}{q_{s^-,j}})\left(\frac{1-\alpha}{\alpha}\right)\end{aligned}$$

and

$$a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} w(k_s + E(d_s)) \\ w(k_{s^-} + E(d_{s^-})) \end{bmatrix}$$

where $a_{11} = \zeta_{\Delta_s q_s}(2\frac{\Phi_{s,j}(2)}{q_{s,j}} + \frac{\Phi_{s,j}(1)}{q_{s,j}})\left(\frac{1-\alpha}{\alpha}\right)$, $a_{12} = \zeta_{\Delta_{s-} q_s}(\frac{\Phi_{s^-,j}(1)}{q_{s,j}} + 2\frac{\Phi_{s^-,j}(2)}{q_{s,j}} + 3\frac{\Phi_{s^-,j}(3)}{q_{s,j}})\left(\frac{1-\alpha}{\alpha}\right)$, $a_{21} = \zeta_{\Delta_s q_{s-}}(\frac{\Phi_{s,j}(1)}{q_{s^-,j}} + 2\frac{\Phi_{s,j}(2)}{q_{s^-,j}})\left(\frac{1-\alpha}{\alpha}\right)$, $a_{22} = \zeta_{\Delta_{s-} q_{s-}}(\frac{\Phi_{s^-,j}(1)}{q_{s^-,j}} + 2\frac{\Phi_{s^-,j}(2)}{q_{s^-,j}} + 3\frac{\Phi_{s^-,j}(3)}{q_{s^-,j}})\left(\frac{1-\alpha}{\alpha}\right)$.

Standard algebra implies that expected value to the providers satisfies

$$\bar{V}_s = \frac{a_1\Gamma_4 - \Gamma_2 a_2}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3} \tag{14}$$

$$\bar{V}_{s^-} = \frac{\Gamma_1 a_2 - a_1\Gamma_3}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3} \tag{15}$$

To solve for the expected profits to the farmer we need to replace the equilibrium value to the provider into its definition,

$$\bar{V}_i = (z\alpha)^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{\alpha}\frac{\Delta_{ij}}{U_i}\right)^{\frac{1-\alpha}{\alpha}} k_i - w(k_i + E(d_i))$$

which yields the expected profits to the farmers as a function of the probabilities of service, and the capacity.

We can solve for the expected profits as

$$U_i = \left(\frac{(z\alpha)^{\frac{1}{\alpha}} k_i}{\bar{V}_i + w(k_i + E(d_i))}\right)^{\alpha/(1-\alpha)} \left(\frac{1-\alpha}{\alpha}\right)\Delta_{ij}$$

Replacing the values of expected profits for the providers,

$$U_s = \left(\frac{(z_s\alpha)^{\frac{1}{\alpha}} k_s}{\left(\frac{a_{11}\Gamma_4 - a_{21}\Gamma_2}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3} + 1\right) w(k_s + E(d_s)) - w(k_{s^-} + E(d_{s^-}))\frac{a_{22}\Gamma_2 - a_{12}\Gamma_4}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}} \right)^{\alpha/(1-\alpha)} \left(\frac{1-\alpha}{\alpha}\right) \Delta_{sj}$$

$$U_{s^-} = \left(\frac{(z_{s^-}\alpha)^{\frac{1}{\alpha}} k_{s^-}}{\left(\frac{a_{22}\Gamma_1 - a_{12}\Gamma_3}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3} + 1\right) w(k_{s^-} + E(d_{s^-})) - w(k_s + E(d_s))\frac{a_{11}\Gamma_3 - a_{21}\Gamma_1}{\Gamma_1\Gamma_4 - \Gamma_2\Gamma_3}} \right)^{\alpha/(1-\alpha)} \left(\frac{1-\alpha}{\alpha}\right) \Delta_{s^-j}$$

Notice that the denominator of the above expression is not the same as the one for the market provider because the probabilities of service are different across providers. This heterogeneity changes the value for the derivatives in Γ. Also, the entries in the a matrix depend on the provider through the probability of service too.

Indifference condition when both providers serve both types of farmers

$$\frac{\Delta_{s,mkt}}{\left((\frac{a_{11}\Gamma_{4j} - a_{21}\Gamma_{2j}}{\Gamma_{1j}\Gamma_{4j} - \Gamma_{2j}\Gamma_{3j}} + 1))w(k_s + E(d_s)) - w(k_{s^-} + E(d_{s^-}))\frac{a_{22}\Gamma_{2j} - a_{12}\Gamma_{4j}}{\Gamma_{1j}\Gamma_{4j} - \Gamma_{2j}\Gamma_{3j}}\right)^{\alpha/(1-\alpha)}}$$
$$= \frac{\Delta_{s,fcfs}}{\left((\frac{a_{11}\Gamma_{4fcfs} - a_{21}\Gamma_{2fcfs}}{\Gamma_{1fcfs}\Gamma_{4fcfs} - \Gamma_{2fcfs}\Gamma_{3fcfs}} + 1))w(k_s + E(d_s)) - w(k_{s^-} + E(d_{s^-}))\frac{a_{22}\Gamma_{2fcfs} - a_{12}\Gamma_{4fcfs}}{\Gamma_{1fcfs}\Gamma_{4fcfs} - \Gamma_{2fcfs}\Gamma_{3fcfs}}\right)^{\alpha/(1-\alpha)}}$$

and an analogous condition for small farmers.

it follows that absent differences in the queue lengths for large and small farmers, and absent differences in transportation costs $E(d_s)$, the value of servicing large orders is higher than the value of servicing small orders (this is because the marginal cost of service is higher for small orders and there are capacity constraints.). *It also follows that $U_{s^-} < U_s$.*. If the probability of servicing one additional small-scale order changes little the probability of service $\tilde{\Delta}_{s^-j}(3) > \tilde{\Delta}_{s^-j}(2)$, the previous inequalities are still true.

The market provider indeed wants to prioritize large scale: Compute $\frac{\partial \Pi_{mkt}}{\partial \chi}$, which are positive given the definitions of $\phi_{i,mkt} \in (0, 1)$ and the value of the provider, equation 9. □

A few characteristics are worth discussing. If the queues have the same lengths and providers can serve at most 2 orders of either type (i.e. there is no capacity advantage for small scale orders) then $\bar{V}_{s^-} < 0$ which would imply that the provider does not want to serve small-scale farmers. By continuity, when providers can serve 3 small-scale order the previous result holds for small probability of service. A higher queue of small-scale requests increases a_2 and potentially the value of serving those farmers.

For the market provider, *t*he value of attracting only large-scale providers is weakly higher than the value of attracting both types of requests. For the market provider, the value of serving both types is smaller than the value of serving just the large farmers as the queue of small-scale orders goes to zero. We want to show that $\bar{V}_s < \bar{V}_{s,only}$. Note that $\tilde{a}$ is the same for large and small scale, and $\lim_{q_{s^-}\to 0} \bar{V}_s < 0$. By continuity, if the queue lenght of small-scale orders is not too large, the market provider prefers to serve large-scale orders only.

4. The rental prices can be computed from the definition of U once the optimal queues have been solved for. Notice that if the queue lengths are the same, then $\tilde{\Delta}_{s,mkt} = \tilde{\Delta}_{s,fcfs}$ which implies $\Delta_{s,mkt} > \Delta_{s,fcfs}$. Because both functions are decreasing in q_s ceteris paribus, it is sufficient that

$$q_{s,mkt} > q_{s,fcfs},$$

to restore equality.

5. To solve for the equilibrium interest rate, we can use *the optimal capital demand* and solve non-linearly for k,

$$k_i = \left(\frac{z_i \alpha}{r_i^{\alpha}}\right)^{1/(1-\alpha)}$$

6. *In equilibrium,* $U_s \geq U_{s^-}$

If the large-scale farmers have weakly lower expected travel costs, $E(d_s) \leq E(d_{s^-})$ the expected profits to the large farmers are higher than for small scale farmers in equilibrium. This is because the provider gets a higher return per hour travelled when serving large farmers, or the marginal cost of servicing a larger farmer is lower. Suppose that the equilibrium is such that $U_s < U_{s^-}$. Then, the expected gain of attracting a large-scale farmer would be higher or equal than that of a low-scale farmer. Therefore the provider could increase profits by lowering the rental price and attracting more large-scale farmers (and therefore increasing expected profits to the large-scale farmer).

7. *The value of serving large farmers is higher than the value of serving small-farmers for any provider* Assume that the queue lengths are identical, then $\bar{V}_s > \bar{V}_{s^-}$ requires (using equation 14)

$$a_1 (\Gamma_4 + \Gamma_3) > a_2 \left(\Gamma_2 + \Gamma_1\right)$$

because when the queue lengths are the same $\Gamma_1 + \Gamma_3 = \Gamma_2 + \Gamma_4$ and $a_1 > a_2$ by definition. Hence, by continuity, if $q_{s,j}, q_{s^-,j}$ are not too different, then the value of serving large farmers is higher than the value of serving small-farmers.

C Numerical Solution and Output

C.1 Value function computation

The value function maps an ordered queue to the expected present value of this queue. Each order i in the queue comprises two dimensions: h_i, the number of hours demanded discretized to 6 bins, and d_i the travel hours to and from the hub that represents a variable cost of service. For a queue length equal to 3, the value function is a mapping from R^6 to R^1.

$$V(\{(k_1, \nu_1), (k_2, \nu_2), (k_3, \nu_3)\}) : R^6 \rightarrow [0, \infty]$$

The relatively high dimensionality of the problem prompts us to implement the sparse-grid method proposed by Smolyak (1963) (see Judd et al. (2014) for details). The grid points are selected for an approximation level of 2, which results in 85 grid points. We then construct a Smolyak polynomial consisting 85 orthogonal basis functions, which belong to the Chebyshev family. The integration nodes are selected by applying the tensor product rule to the one-dimensional Smolyak grid points at the approximation level of 2. Integration is carried out using Newton-Cotes quadrature.

C.2 Simulations

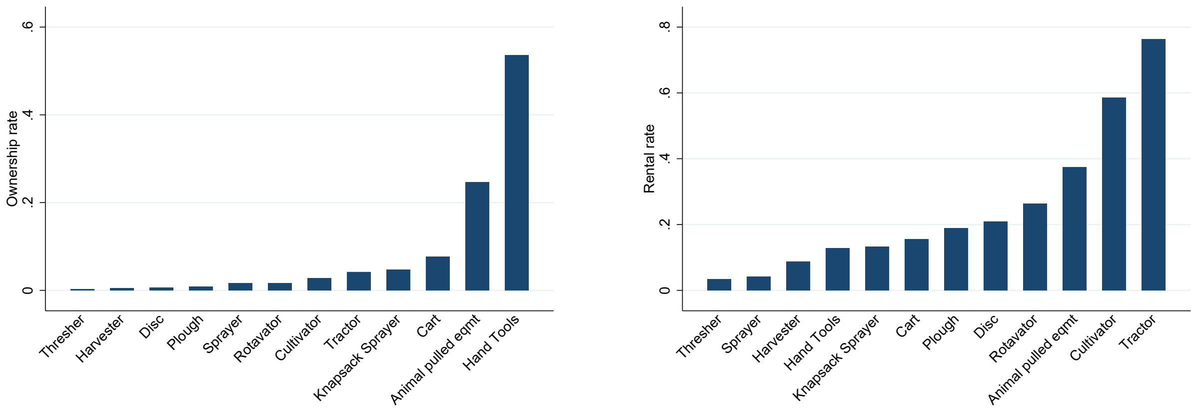
We simulate the expected wait time and productivity cost under the FCFS arrangement and the market arrangement respectively for three cases: when productivity is uncorrelated, negatively correlated or positively correlated to the number of hours demanded. Productivity, measured in revenue per acre, is simulated and assigned to each order observed in the actual data. We make a large number of draws of productivity sequences, each with length equal to the number of actual orders, from a log normal distribution where the parameters are obtained by fitting the actual productivity information to a log normal distribution by hub. We then choose a sequence for each simulation case that produces a correlation with hours demanded in the data that is the closest to a target correlation for that case, and assign that sequence to the actual orders. In the uncorrelated case, the target is zero; in the negatively correlated case, the target is the actual correlation for that hub; in the positively correlated case, the target is symmetric to the negative correlation.

We use bootstrap sampling of the actual orders for the simulation and assume each bootstrap sample represents an actual queue.

We compute the wait time for the first three orders in each bootstrap sample under the FCFS arrangement and the market arrangement respectively. In any period if one or more out of these three orders are not served, the queue is filled going through the bootstrap sample. To avoid having a large order "jamming" the queue, we assume every order is feasible. This implies that the maximum wait time under the FCFS arrangement is 3. We cap the wait time at 5 for the market arrangement. The productivity cost is then calculated by multiplying the simulated productivity by a percentage loss as described in the table 4.

D Additional Tables and Figures

Figure 11: Ownership and rentals by implement.



Owned Rentals

The ownership (rental) rate is the share of farmers that report to own a given implement relative to the total population surveyed.

Figure 12: Value Functions